The Effect of Internal Heat Generation of a Reactive Hydromagnetic (MHD) Fluid Flow through a Vertical Plate

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Abstract
In this study, we discussed the flow of an electrically conducting fluid with internal heat generation over a vertical plate on a reactive MHD fluid. We obtained the semi analytical solutions of the dimensionless equations governing the fluid flow using the method of weighted residual by the collocation method. The parameters with their effect on velocity and temperature fields are discussed graphically using Mapple 17 computer software. We showed that the internal heat generation, magnetic field intensityand the reactive rate have significant effects on the velocity profiles and temperature distributions.

Key words: Magnetohydrodynamics (MHD), vertical plate, reactive fluid, internal heat generation.

Introduction

Studies relating to flow of fluids between two parallel plates are on the increase in recent time due to its numerous applications in many industrial and engineering processes like in accelerators, aerodynamic heating, polymer technology, purification of crude oil and magnetohydrodynamics (MHD) generators. The study of heat generation or absorption effects in moving fluids is important in view of several physical problems, such as fluids undergoing exothermic and endothermic chemical reactions (Hassan and Gbadeyan, 2015).

Reactive hydro- magnetic fluid flows are often accompanied with heat transfer in many industrial and engineering applications. For instance, Makinde and Beg (2010) devoted their study to investigate the inherent irreversibility and thermal stability in a reactive electrically conducting fluid flowing steadily through a channel with isothermal walls under the influence of a transversely imposed magnetic field. Recently, Hassan and Gbadeyan (2015) investigated the entropy generation analysis of a reactive hydro-magnetic fluid flow through a channel with isothermal wall temperature under different chemical kinetics without taking into account the effects of internal heat generation within the flow system.

Lot of research works concerning the fluid flow between two vertical plates has been obtained under different physical and chemical conditions. Possible heat generation effects may alter the temperature distribution as well as the particle deposition rate of the fluid flow (Jha and Ajibade, 2009; Makinde and Beg, 2010; Hassan and Fenuga, 2011; Adesanya and Makinde, 2011; Hassan and Gbadeyan, 2015), this informed our study of the effects of internal heat generation within the flow system.

It is well known that the rate of heat transfer is temperature dependent, which increases the interaction of moving fluid and thus influence the internal energy of the flow regime. This interaction according to Frank-Kamenettski (1969), Makinde and Beg (2010) as well as Hassan and Gbadeyan (2014)bring about the condition of thermal runaway or ignition in the flow system to predict critical and unsafe situations.

The weighted residual method may be considered to be a unified version of a group of methods used to solve approximately boundary value, initial value and Eigen value problems. There are some simple and accurate approximation techniques for solving differential equations called the Weighted Residuals Methods (WRMs). Collocation, Galerkin and Least Square are examples of the WRMs. Stern and Rasmussen (1996) used collocation method for solving a third order linear differential equation. Vaferi *et al*. (2012) studied the feasibility of applying Orthogonal Collocation method to solve diffusivity equation in the radial transient flow system. Hendi and Albugami (2010) used Collocation and Galerkin methods for solving Fredholm–Volterra integral equation. Recently least square method was introduced by Aziz and Bouaziz (2011) and applied for it in predicting the performance of a longitudinal fin. They found that least squares method is simple compared with other analytical methods. Shaoqin and Huoyuan (2008) developed and analyzed least-squares approximations for the incompressible magneto-hydrodynamic equations. Recently, Hatami *et al*. (2014) studied the third grade non-Newtonian blood conveying gold nanoparticles in a porous and hollow vessel by two analytical methods called Least Square Method (LSM) and Galerkin Method (GM).

Hence, in the present study we will use the weighted residual method by collocation method to discuss an electrically conducting fluid with internal heat generation over a vertical plate on a reactive MHD fluid.

Mathematical Formulation

Consider a steady flow of an incompressible viscous fluid flow between two vertical Channelsof a reactive MHD flow withinternal heat generation. Let the *x*-axis be directed upward along the plate and the y-axis normal to the plate. The flow is electrically conducting in the presence of an applied magnetic field and is maintained due to a constant pressure gradient.

Let *u* and *v* be the velocity components along the *x*- and *y*- axes respectively. The continuity equation is given as

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \qquad\qquad (1)$$

If the flow is in the x-direction, $\frac{\partial \bar{u}}{\partial x} = 0$, which implies $\frac{\partial \bar{v}}{\partial y} = 0$ Then the steady equations of motion (momentum and energy equation) for the system is presented by

$$-\frac{\partial \bar{p}}{\partial \bar{x}} + v\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\sigma B_0^2 u}{\rho} + g\beta(T - T_0) = 0 \quad (2)$$

$$\alpha \frac{\partial^2 T}{\partial \bar{y}^2} + Ra - \lambda(T - T_0) = 0 \qquad (3)$$

Where α is the thermal diffusivity, v is the kinematic viscosity, β is the volumetric expansion coefficient for heat transfer, ρ is the density, g is the acceleration due to gravity, T is the temperature, T_0 is the reference or ambient temperature, λ is the heat generation coefficient and Ra is the reaction term

With the following boundary conditions

$$\bar{u}(0) = u_0, \quad \bar{u}(L) = 0. \qquad (4)$$

$$T(0) = T(L) = T_0 \qquad (5)$$

L is the width of the channel and u_0 is the velocity of the left plate .

Dimensional Analysis

By the scaling principle, we present the following dimensionless scales

$$u = \frac{\bar{u}}{u_0}, y = \frac{\bar{y}}{L}, Ra = QC_0A\ell^{-\frac{E}{RT}} \text{ and } \theta = \frac{T - T_0}{\varepsilon T_0} \text{ (6)}$$

And reduced the dimensioned equations into dimensionless momentum and energy equations as

$$P + \frac{\partial^2 u}{\partial y^2} - Nu + Gr\theta = 0 \qquad (7)$$

$$\frac{\partial^2 \theta}{\partial y^2} - \lambda_0\theta + \lambda_1\ell^{\frac{\theta}{(\varepsilon\theta+1)}} = 0 \qquad (8)$$

With the following boundary conditions

$$u(0) = 1, u(1) = 0 \qquad (9)$$

$$\theta(0) = \theta(1) = 0 \qquad (10)$$

Where, N is the Hartman Number, Gr is the Grash of Number, P is the Pressure force, λ_0 is the Internal heat generation and λ_1 is the Arrhenius term.

Weighted Residual Method (WRM) by Collocation method

In this method, the weighting functions are taken from the family of Dirac δ functions in the domain. That is, $W_i(y) = \delta(y - y_i)$. The Dirac δ function has the property that

$$\delta(y - y_i) = \begin{cases} 1, & x = x_i \\ 0, & otherwise \end{cases} \qquad (11)$$

The idea in the **WRM** is to force the residual to zero in some average sense over the domain. That is

$$\int_Y R(y)W_i dy = 0, \quad i = 1,2,\cdots,n \qquad (12)$$

where the number of weight functions W_i is exactly equal the number of unknown constants a_i in $\tilde{u}(y)$. The result is a set of n algebraic equations for the unknown constants a_i (Salih, 2016).

the fixed points $y_i \in [a,b], (i = 1,2,\cdots,n)$ are called collocation points. The number of collocation points selected must be equal to the number of unknown coefficients $a_i's$ in the definition of approximating function, $u(y)$.

Hence, the integration of the weighted residual statement results in the forcing of the residual to zero at specific points in the domain. That is, integration of (12) with $W_i(y) = \delta(y - y_i)$ results in $R(y_i) = 0$

Thus, by the collocation method, we will use a polynomial function as a basis. That is, let the approximating function $u(y)$ which can be chosen arbitrary be

$$\left.\begin{array}{l} u(y) = a_0 + a_1 y + a_2 y^2 + a_3 y^3 + a_4 y^4 \\ \theta(y) = b_0 + b_1 y + b_2 y^2 + b_3 y^3 + b_4 y^4 \end{array}\right\} (13)$$

Solution to the dimensionless momentum and energy equations using the collocation method

We will chose the trial functions in such a way that the assumed function $\tilde{u}(y)$ satisfies the global boundary conditions for (13) Thus,

$$u(y) = 1 - y^4 + a_1 y(1 - y^3)$$
$$+ a_2 y^2(1 - y^2) + a_3 y^3(1 - y) \qquad (14)$$

and

$$\theta(y) = b_1 y(1 - y^3) + b_2 y^2(1 - y^2) + b_3 y^3(1 - y) \qquad (15)$$

Hence,

$$R_1(y) = P + \left(\begin{array}{c} -12y^2 - 12a_1 y^2 + 2a_2(1 - 12y^2) \\ + 6a_3 y(1 - 2y) \end{array} \right)$$
$$- N \left(\begin{array}{c} 1 - y^4 + a_1 y(1 - y^3) + a_2 y^2(1 - y^2) \\ + a_3 y^3(1 - y) \end{array} \right) \qquad 16$$
$$+ Gr \left(\begin{array}{c} b_1 y(1 - y^3) + b_2 y^2(1 - y^2) \\ + b_3 y^3(1 - y) \end{array} \right)$$

$$R_2(y) = \left(\begin{array}{c} -12b_1 y^2 + 2b_2(1 - 6y^2) \\ + 6b_3 y(1 - 2y) \end{array} \right)$$
$$- \lambda_0 \left(\begin{array}{c} b_1 y(1 - y^3) + b_2 y^2(1 - y^2) \\ + b_3 y^3(1 - y) \end{array} \right) \qquad 17$$
$$+ \lambda_1 \ell^{\frac{(b_1 y(1-y^3) + b_2 y^2(1-y^2) + b_3 y^3(1-y))}{(\varepsilon(b_1 y(1-y^3) + b_2 y^2(1-y^2) + b_3 y^3(1-y)) + 1)}}$$

Evaluating equations (16) and (17) at the following collocation points $(y = 0.25, y = 0.5, y = 0.75)$ gives the following relations.

$$R_1(0.25) = P - 0.7500a_1 + 1.2500a_2 + 0.7500a_3 - 0.7500$$
$$- N \left(\begin{array}{c} 0.99609375 + 0.24609375a_1 \\ + 0.05859375a_2 + 0.01171875a_3 \end{array} \right)$$
$$+ Gr \left(\begin{array}{c} 0.24609375b_1 + 0.05859375b_2 \\ + 0.01171875b_3 \end{array} \right) = 0$$
$$R_2(0.25) = -0.7500b_1 + 1.2500b_2 + 0.7500b_3$$
$$- \lambda_0 \left(\begin{array}{c} 0.24609375b_1 + 0.05859375b_2 \\ + 0.01171875b_3 \end{array} \right) \qquad 18$$
$$+ \lambda_1 e^{\frac{(0.24609375b_1 + 0.05859375b_2 + 0.01171875b_3)}{\varepsilon(0.24609375b_1 + 0.05859375b_2 + 0.01171875b_3) + 1}} = 0$$

$$
\left.
\begin{aligned}
R_1(0.5) &= P - 3.00a_1 - 1.00a_2 - 3.00 \\
&\quad -N\left(\begin{array}{l} 0.9375 + 0.4375a_1 + 0.1875a_2 \\ + 0.0625a_3 \end{array}\right) \\
&\quad +Gr\left(\begin{array}{l} 0.4375b_1 + 0.1875b_2 \\ + 0.0625b_3 \end{array}\right) = 0 \\
R_2(0.5) &= -3.00b_1 - 1.00b_2 \\
&\quad -\lambda_0\left(0.4375b_1 + 0.1875b_2 + 0.0625b_3\right) \\
&\quad +\lambda_1 e^{\frac{(0.4375b_1 + 0.1875b_2 + 0.0625b_3)}{\varepsilon(0.4375b_1 + 0.1875b_2 + 0.0625b_3) + 1}} = 0
\end{aligned}
\right\}
\qquad 19
$$

$$
\left.
\begin{aligned}
R_1(0.75) &= P - 6.7500a_1 - 4.7500a_2 \\
&\quad -2.2500a_3 - 6.7500 \\
&\quad -N\left(\begin{array}{l} 0.68359375 + 0.43359375a_1 \\ + 0.24609375a_2 + 0.10546875a_3 \end{array}\right) \\
&\quad +Gr\left(\begin{array}{l} 0.43359375b_1 + 0.24609375b_2 \\ + 0.10546875b_3 \end{array}\right) = 0 \\
R_2(0.75) &= -6.7500b_1 - 4.7500b_2 - 2.2500b_3 \\
&\quad -\lambda_0\left(\begin{array}{l} 0.43359375b_1 + 0.24609375b_2 \\ + 0.10546875b_3 \end{array}\right) \\
&\quad +\lambda_1 e^{\frac{(0.43359375b_1 + 0.24609375b_2 + 0.10546875b_3)}{\varepsilon(0.43359375b_1 + 0.24609375b_2 + 0.10546875b_3) + 1}} = 0
\end{aligned}
\right\}
\qquad 20
$$

The a_i's and b_i's are obtained from solving (18), (19) and (20) as a system of linear equations with suitable values of the dimensionless constants and with the help of maple 17 software we summarize the solution in the table together with the numerical solution of the momentum and energy equations as follows.

Table1: Coefficients of the Basis Functions with Increasing Internal Heat Generation λ_0 and $\left(-P=1, N=0.2\right)$, $\left(Gr=0.1, \varepsilon=0.5 \, and \, \lambda_1 = 1.2\right)$

λ_0	a_1	a_2	a_3	b_1	b_2	b_3
1.2	-1.553547322	0.5994305394	-0.05864014171	0.5051908141	-0.5901416525	0.1699016768
1.4	-1.553609463	0.5994166552	-0.05848808951	0.4985351037	-0.5895618927	0.1820535781
1.6	-1.553669635	0.5994033487	-0.05834113393	0.4920813209	-0.5889325390	0.1937024362

Table 2: Coefficients of the Basis Functions with Increasing Reactive Rate λ_1 and $\left(-P=1, N=0.2, Gr=0.1\right)$, $\left(\varepsilon=0.5 \, and \, \lambda_0 = 1.2\right)$

λ_1	a_1	a_2	a_3	b_1	b_2	b_3
1.2	-1.553547322	0.5994305394	-0.05864014171	0.5051908141	-0.5901416525	0.1699016768
1.4	-1.552936843	0.5993847083	-0.05976943689	0.5827246268	-0.6861007858	0.2067523180
1.6	-1.552340174	0.5993371739	-0.06086770593	0.6586871810	-0.7812463833	0.2451184046

Table 3: Coefficients of the Basis Functions with Varying Hartman Number N and $(-P = 1, \lambda_1 = 1.2, Gr = 0.1)$, $(\varepsilon = 0.5 \, and \, \lambda_0 = 1.2)$

N	a_1	a_2	a_3	b_1	b_2	b_3
1	-1.552695423	0.5996078885	-0.06069863735	0.5972991736	-0.6045225430	0.01444673890
2	-2.012818193	1.464122307	-0.5888827392	0.5972991736	-0.6045225430	0.01444673890
3	-2.231801428	1.919487300	-0.9182288879	0.5972991736	-0.6045225430	0.01444673890

Table 4: Coefficients of the Basis Functions with Varying Pressure P and $(N = 0.2, \lambda_1 = 1.2, Gr = 0.1)$, $(\varepsilon = 0.5 \, and \, \lambda_0 = 1.2)$

P	a_1	a_2	a_3	b_1	b_2	b_3
1	-1.769224197	0.9910133792	-0.2819622032	0.5999624688	-0.6002005634	0.0004761892805
2.5	-2.462201562	1.739707517	-0.3933957494	0.5999624688	-0.6002005634	0.0004761892805
5	-3.617163837	2.987531081	-0.5791183263	0.5999624688	-0.6002005634	0.0004761892805

Table 5: Coefficients of the Basis Functions with Varying Grashorf Number Gr and $(N = 0.2, \lambda_1 = 1.2, P = 1)$, $(\varepsilon = 0.5 \, and \, \lambda_0 = 1.2)$

Gr	a_1	a_2	a_3	b_1	b_2	b_3
0.1	-1.769224197	0.9910133792	-0.2819622032	0.5999624688	-0.6002005634	0.0004761892805
0.5	-1.750979201	09905933739	-0.3176121851	0.5999624688	-0.6002005634	0.0004761892805
1	-1.728172955	0.9900683672	-0.3621746624	0.5999624688	-0.6002005634	0.0004761892805

Thus, by the collocation method, solution of the momentum and energy equations are given by

$$u_{\lambda_0 = 1.2}(y) = (1 - y^4) - 1.553547322 \, y(1 - y^3)$$
$$+ 0.5994305394 \, y^2(1 - y^2) \tag{21}$$
$$- 0.05864014171 \, y^3(1 - y)$$

$$u_{\lambda_0 = 1.4}(y) = (1 - y^4) - 1.553609463 \, y(1 - y^3)$$
$$+ 0.5994166552 \, y^2(1 - y^2) \tag{22}$$
$$- 0.05848808951 \, y^3(1 - y)$$

$$u_{\lambda_0 = 1.6}(y) = (1 - y^4) - 1.553669635 \, y(1 - y^3)$$
$$+ 0.5994033487 \, y^2(1 - y^2) \tag{23}$$
$$- 0.05834113393 \, y^3(1 - y)$$

and

$$\theta_{\lambda_0 = 1.2}(y) = 0.5051908141 \, y(1 - y^3)$$
$$- 0.5901416525 \, y^2(1 - y^2) \tag{24}$$
$$+ 0.1699016758 \, y^3(1 - y)$$

$$\theta_{\lambda_0=1.4}(y) = 0.4985351087y(1-y^3)$$
$$-0.5895618927y^2(1-y^2) \tag{25}$$
$$+0.1820535781y^3(1-y)$$

$$\theta_{\lambda_0=1.6}(y) = 0.4920813209y(1-y^3)$$
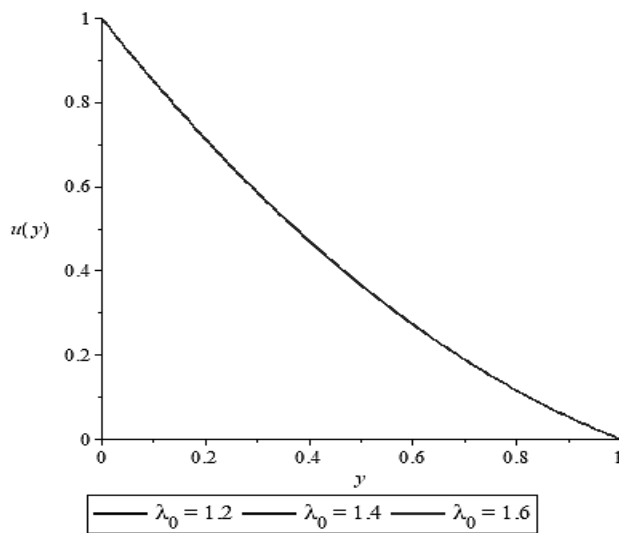$$-0.5889325390y^2(1-y^2) \tag{26}$$
$$+0.1937024362y^3(1-y)$$

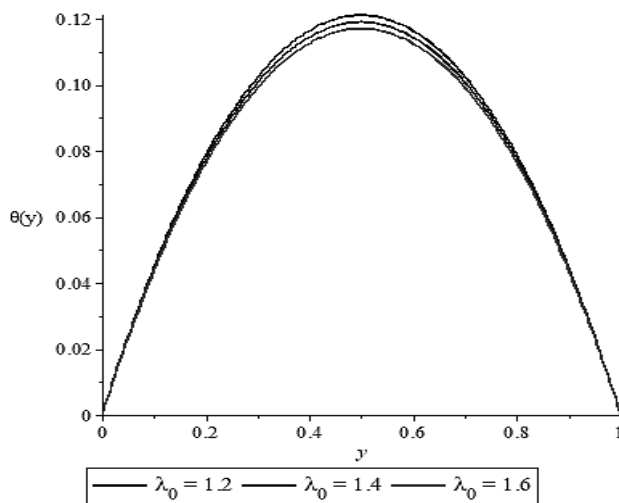Figure 1: Effect of the Increasing Internal HeatGeneration λ_0 on the Velocity Profile.

Figure 2: Effect of the Increasing Internal Heat Generation λ_0 on the Temperature profile.

$$u_{\lambda_1=1.2}(y) = (1-y^4) - 1.553547322y(1-y^3)$$
$$+0.5994305394y^2(1-y^2) \tag{27}$$
$$-0.0586401471y^3(1-y)$$

$$u_{\lambda_1=1.4}(y) = (1-y^4) - 1.5529368483y(1-y^3)$$
$$+0.5993847083y^2(1-y^2)$$
$$-0.05976943689y^3(1-y) \tag{28}$$

$$u_{\lambda_1=1.6}(y) = (1-y^4) - 1.5523401774y(1-y^3)$$
$$+0.5993371739y^2(1-y^2)$$
$$-0.06086770593y^3(1-y) \tag{29}$$

and

$$\theta_{\lambda_1=1.2}(y) = 0.5051908141y(1-y^3)$$
$$-0.5901416525y^2(1-y^2)$$
$$+0.1699016768y^3(1-y) \tag{30}$$

$$\theta_{\lambda_1=1.4}(y) = 0.5827246268y(1-y^3)$$
$$-0.6861007858y^2(1-y^2)$$
$$+0.2067523180y^3(1-y) \tag{31}$$

$$\theta_{\lambda_1=1.6}(y) = 0.6586871810y(1-y^3)$$
$$-0.7812463883y^2(1-y^2)$$
$$+0.2451184046y^3(1-y) \tag{32}$$
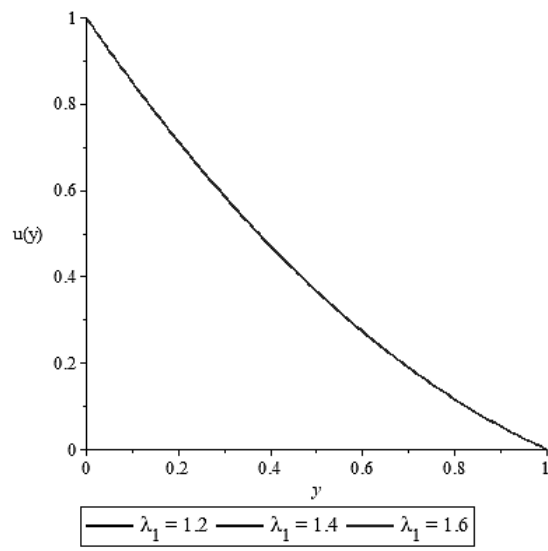
Figure 3: Effect of the Reactive Rate λ_1 on the velocity Profile.
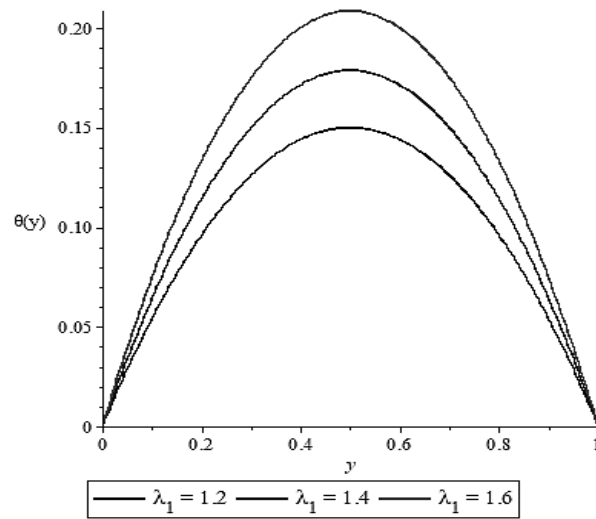
Figure 4: Effect of the Reactive Rate λ_1 on the TemperatureProfile

$$u_{N=1}(y) = \left(1 - y^4\right) - 1.55269542 3 y\left(1 - y^3\right)$$
$$+ 0.5996078885 y^2 \left(1 - y^2\right)$$
$$- 0.06069863735 y^3 \left(1 - y\right) \tag{33}$$

$$u_{N=2}(y) = \left(1 - y^4\right) - 2.01281819 3 y\left(1 - y^3\right)$$
$$+ 1.46412230 7 y^2 \left(1 - y^2\right)$$
$$- 0.5888827392 y^3 \left(1 - y\right) \tag{34}$$

$$u_{N=3}(y) = \left(1 - y^4\right) - 2.23180142 3 y\left(1 - y^3\right)$$
$$+ 1.91948730 0 y^2 \left(1 - y^2\right)$$
$$- 0.9182288879 y^3 \left(1 - y\right) \tag{35}$$

and

$$\theta_{N=1}(y) = 0.5972991736 y\left(1 - y^3\right)$$
$$- 0.6045225480 y^2 \left(1 - y^2\right)$$
$$+ 0.0144467389 0 y^3 \left(1 - y\right) \tag{36}$$

$$\theta_{N=2}(y) = 0.5972991736 y\left(1 - y^3\right)$$
$$- 0.6045225480 y^2 \left(1 - y^2\right)$$
$$+ 0.0144467389 0 y^3 \left(1 - y\right) \tag{37}$$

$$\theta_{N=3}(y) = 0.5972991736 y\left(1 - y^3\right)$$
$$- 0.6045225480 y^2 \left(1 - y^2\right)$$
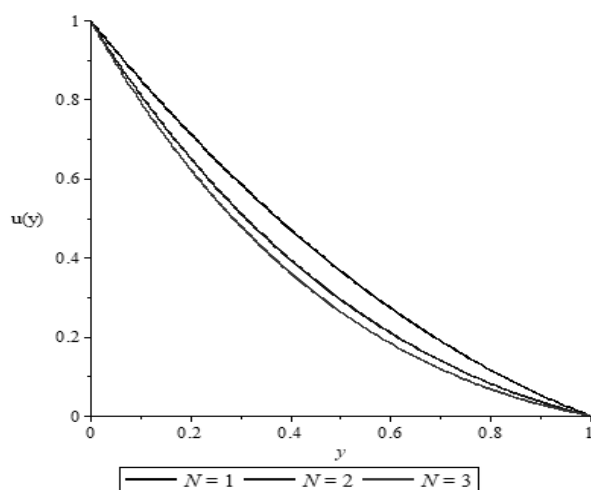$$+ 0.0144467389 0 y^3 \left(1 - y\right) \tag{38}$$

Figure 5: Effect of the Hartman Number N onthe Velocity Profile
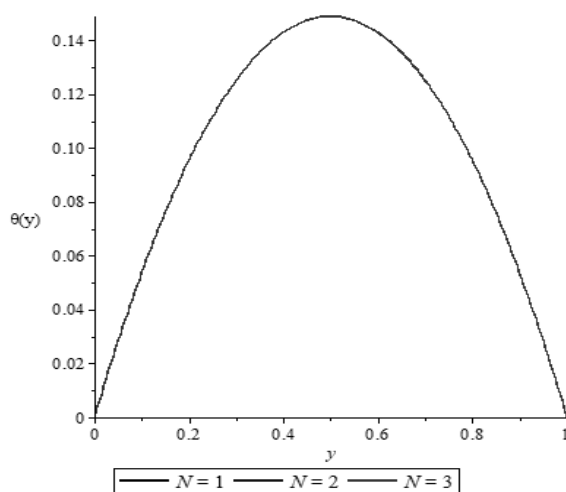
Figure 6: Effect of the Hartman Number N on the Temperature Profile

$$u_{P=1}(y) = \left(1 - y^4\right) - 1.769224197\, y\left(1 - y^3\right)$$
$$+0.991013379 2 y^2\left(1 - y^2\right) \tag{39}$$
$$-0.281962203 2 y^3\left(1 - y\right)$$

$$u_{P=2.5}(y) = \left(1 - y^4\right) - 2.462201562\, y\left(1 - y^3\right)$$
$$+1.739707517\, y^2\left(1 - y^2\right) \tag{40}$$
$$-0.393395749 4 y^3\left(1 - y\right)$$

$$u_{P=5}(y) = \left(1 - y^4\right) - 3.617163837\, y\left(1 - y^3\right)$$
$$+2.987531081\, y^2\left(1 - y^2\right) \tag{41}$$
$$-0.579118326 3 y^3\left(1 - y\right)$$

and

$$\theta_{P=1}(y) = 0.599962468 8 y\left(1 - y^3\right)$$
$$-0.600200568 4 y^2\left(1 - y^2\right) \tag{42}$$
$$+0.000476189 2805 y^3\left(1 - y\right)$$

$$\theta_{P=2.5}(y) = 0.5999624688y\left(1 - y^3\right)$$
$$-0.6002005664y^2\left(1 - y^2\right)$$
$$+0.0004761892805y^3\left(1 - y\right)$$

(43)

$$\theta_{P=5}(y) = 0.5999624688y\left(1 - y^3\right)$$
$$-0.6002005664y^2\left(1 - y^2\right)$$
$$+0.0004761892805y^3\left(1 - y\right)$$

(44)

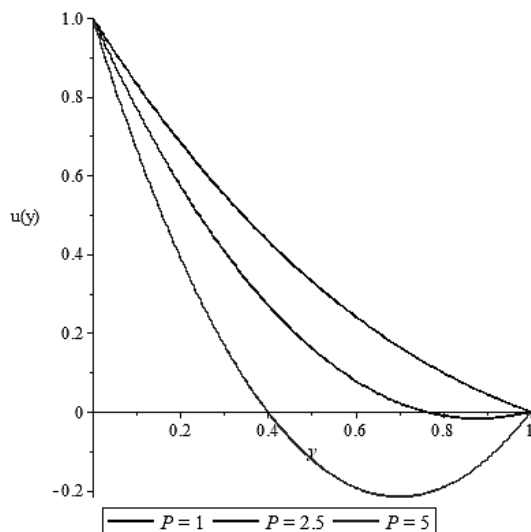**Figure 7:** Effect of the Pressure P on the Velocity Profile
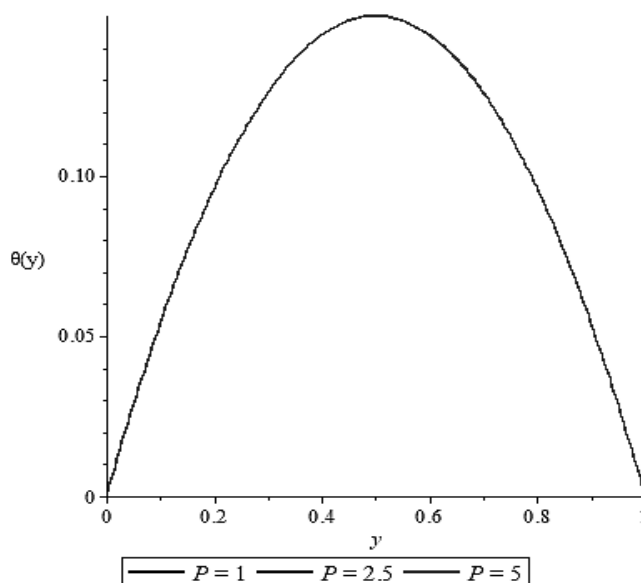
Figure 8: Effect of the Pressure P on the TemperatureProfile

$$u_{Gr=0.1}(y) = \left(1 - y^4\right) - 1.7692241197y\left(1 - y^3\right)$$
$$+0.9910133792y^2\left(1 - y^2\right)$$
$$-0.2819622032y^3\left(1 - y\right)$$

(45)

$$u_{Gr=0.5}(y) = \left(1 - y^4\right) - 1.75097920 1\, y\left(1 - y^3\right)$$
$$+ 0990593373 9\, y^2\left(1 - y^2\right) \tag{46}$$
$$- 0.317612185 1\, y^3\left(1 - y\right)$$

$$u_{Gr=1}(y) = \left(1 - y^4\right) - 1.72817295 5\, y\left(1 - y^3\right)$$
$$+ 0.990068367 2\, y^2\left(1 - y^2\right) \tag{47}$$
$$- 0.362174662 4\, y^3\left(1 - y\right)$$

and

$$\theta_{Gr=0.1}(y) = 0.599962468 8\, y\left(1 - y^3\right)$$
$$- 0.600200563 4\, y^2\left(1 - y^2\right) \tag{48}$$
$$+ 0.000476189 2805\, y^3\left(1 - y\right)$$

$$\theta_{Gr=0.5}(y) = 0.599962468 8\, y\left(1 - y^3\right)$$
$$- 0.600200563 4\, y^2\left(1 - y^2\right) \tag{49}$$
$$+ 0.000476189 2805\, y^3\left(1 - y\right)$$

$$\theta_{Gr=1}(y) = 0.599962468 8\, y\left(1 - y^3\right)$$
$$- 0.600200563 4\, y^2\left(1 - y^2\right) \tag{50}$$
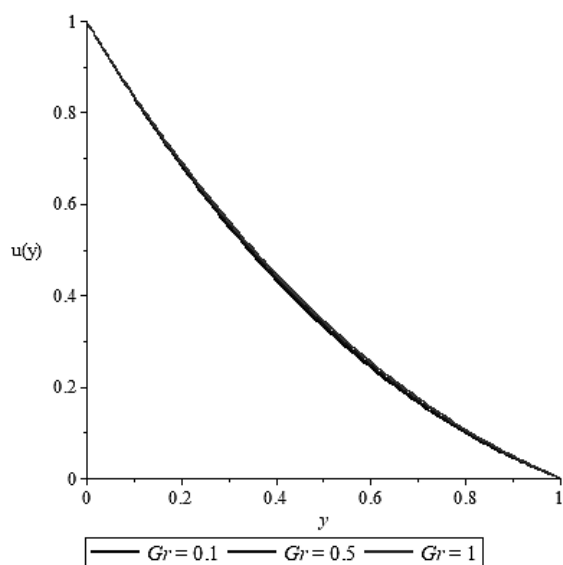$$+ 0.000476189 2805\, y^3\left(1 - y\right)$$

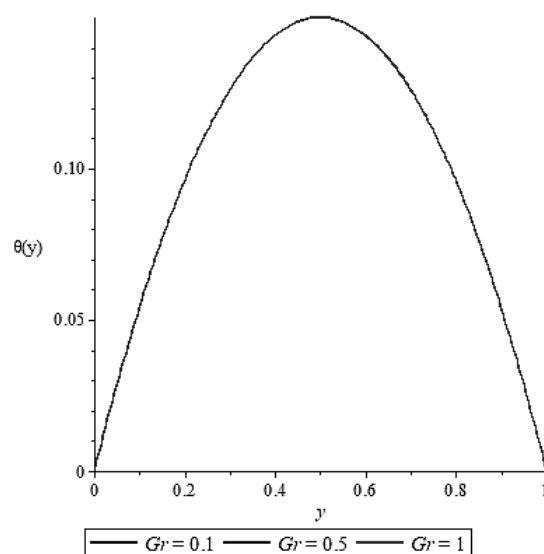**Figure 9:** Effect of the Grashorf Number Gr onthe VelocityProfile

Figure 10: Effect of the Grashorf Number Gr on the Temperature Profile

Discussion

We will discuss the effects of the internal heat generation together with other important flow parameters on the hydromagnetic fluid flow between two vertical plates with an isothermal wall temperature in this session. These are presented in tables and graphs using Maple 17 Mathematical software.

Figure 1 and 2 shows the effect of the increasing internal heat generation λ_0 on the velocity and temperature profile respectively. Increase in the internal heat generation parameter λ_0 reduces the temperature of the fluid and has no significant effect on the velocity profile while increase in the Reactive Rate λ_1 increases the temperature and shows no significant change in the velocity profile as can be seen by Figure 3 and 4. Increase in the Stuart Number N reduces the velocity of the flow. The Stuart Number N which is also known as the magnetic interaction parameter which is the ratio of electromagnetic to inertial forces on flow have no significant effect on the temperature profile as can be seen from figures 5 and 6 respectively. Figures 7 and 8 shows that the pressure forces P which drives the flow also reduces the velocity and even goes beyond the boundary line at higher pressure forces while the temperature has no significant change with increase in the pressure forces. The Grashof Number Gr which is a dimensionless number in fluid dynamics and heat transfer approximates the ratio of the buoyancy to the viscous force acting on a fluid, increases with increase in the velocity profile. It however does not affect the temperature as seen by figures 9 and 10.

Conclusion

The effect of internal heat generation on a steady hydromagnetic fluid flow between two Channels is investigated. Analytical solutions of the equations governing the fluid flow are obtained using the method of weighted residual by collocation method. In view of the above results, the following are the major contributions in this study:

i. Theeffect of the increasing Stuart Number N and the pressure P reduces the velocity while the increasing Grashorf Number Gr increases the fluid velocity and the temperature remains indifferent.

ii. Due to collision of the fluid particles, internal heat λ_0 is being generated which eventually leads to a decrease in the fluid temperature.

iii. The temperature increases as the reactive rate λ_1 which is the modification of the Arrhenius Number increases while the velocity remains indifferent.

Finally, we showed that the internal heat generation, magnetic field intensity and the reactive rate have significant effects on the velocity profiles and temperature distributions.

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