

Simulation of Electron-Electron Two Stream Instability (ETSI)

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Abstract

Stream instabilities are widely studied due to their importance in understanding astrophysical phenomena such as acceleration of high velocity of solar wind. In this work, the simulation of electron two stream instability was performed using Vorp Simulation (VSim) code to explore the kinetic energy of plasma that arises due to the interaction between two counter-streaming electron beams at different velocities as well as different electron densities. The electron beam velocity was varied in the range of 3.58×10^6 m/s - 7.98×10^6 m/s and the resulting kinetic energy of plasma increased from 19×10^{-6} J - 210×10^{-6} J respectively. Also, increasing the electron density at fixed beam velocity from 1.05×10^{14} m⁻³ - 5.84×10^{14} m⁻³, the kinetic energy was observed to increase from 100×10^{-6} J - 200×10^{-6} J. However, the kinetic energy of the electron increases more with increasing beam velocity than with increasing electron density. The electric field energy which arose due to the interaction of the streaming beams did not exceed the energy of the beams.

Key words: Two Stream Instability, Plasma oscillation, Vorp Simulation Code, Kinetic Energy.

Introduction

One of the important collective interactions in a plasma is the two-stream instability (Fanchenko *et al.*, 1964). Plasma instability that is caused by an energetic particle stream injected into a plasma, or setting a current along the plasma where different species (ions and electrons) can have different drift velocities (Mohammad *et al.*, 2014) which generally go by the cognomen two stream instability is a well know phenomena (Ben *et al.*, 1990) in the context of fusion plasmas, space plasmas, and high-energy accelerators (Marileni *et al.*, 2013). When two-streams of electron beams move through each other, a density perturbation is created because of the reinforcing of the charge particles of one beam by the forces due to bunching of the other beam and vice versa. The density perturbation generates an electric field which can grow exponentially. The cause of this instability is thought of as originating from a point source disturbance within two-beam plasma. If a density fluctuation arises from this disturbance in one stream of particles, then the electric field will initiate a plasma oscillation at that location. However, these fields can modulate the electron densities of the second stream and the drift of these density modulations through each other can result in energy exchange (King *et al.*, 2011).

The two stream instabilities are the most common instabilities in plasmas. Usually plasma consists of two or more kinds of particles moving with different velocities. Under suitable conditions the two-stream instability can occur and particle energy is transferred into the energy of plasma wave excitation (Martin *et al.*, 2011).

Basic equations

The relevant equations describing the interaction between the electron plasma wave and the plasma electrons are the equations of motion and continuity for the electron fluid together with Poisson's equation, which provides the self-consistent coupling between the electrostatic wave and the electron motion. Since the electron velocity, the electrostatic field as well as the propagation vector all lie in the x direction, the problem is truly one-dimension and the governing equations read.

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} = -\frac{e}{m}E \quad 1$$

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nv) = 0 \quad 2$$

$$\frac{\partial E}{\partial x} = -\frac{e}{\epsilon_0}(n - n_0) \quad 3$$

where v and n denote the velocity and density, respectively, of the electron fluid and E is the electrostatic electric field. Furthermore, n_0 is the constant density of the neutralizing ion background, e and m are the electron charge and mass, respectively, and ϵ_0 is the dielectric constant in vacuum.

Mathematical Formulation of Plasma Oscillation

In order to analyze the characteristic propagation properties of the electron plasma wave, one considers the evolution of small perturbations of density, velocity and electric field on a stationary and homogeneous background according to;

$$v = v_0 + v_1 \quad 4$$

$$n = n_0 + n_1 \quad 5$$

$$E = E_0 + E_1, \quad 6$$

where index 0 denotes a stationary and homogeneous equilibrium quantity and index 1 denotes a small perturbation around the corresponding equilibrium quantity.

If consideration is given to a situation where the equilibrium electric field is zero, i.e., $E_0 = 0$, and assuming that the background electrons are stationary with respect to the background ions, i.e., $v_0 = 0$. Then substitution of Eqs. (1), (2) and (3) into Eqs. (4), (5) and (6) respectively and linearizing, i.e., keeping only terms linear in the small wave quantities, gives

$$\frac{\partial v_1}{\partial t} = -\frac{e}{m}E_1 \quad 7$$

$$\frac{\partial n_1}{\partial t} + n_0 \frac{\partial v_1}{\partial x} = 0 \quad 8$$

$$\frac{\partial E_1}{\partial x} = -\frac{e}{\epsilon_0}n_1 \quad 9$$

Looking for travelling plane wave solutions proportional to $e^{i(kx - \omega t)}$ and with constant amplitude, where ω and k are the frequency and wave number, respectively, of the electron plasma wave, Eqs. (7), (8) and (9) can be written as

$$-i\omega v_1 = -\frac{e}{m}E_1 \tag{10}$$

$$-i\omega n_1 + in_0kv_1 = 0 \tag{11}$$

$$ikE_1 = -\frac{e}{\epsilon_0}n_1 \tag{12}$$

$$-i(\omega - kv_0)v_1 = -\frac{e}{m}E_1 \tag{17}$$

$$-i(\omega - kv_0)n_1 + in_0kv_1 = 0 \tag{18}$$

$$ikE_1 = -\frac{e}{\epsilon_0}n_1 \tag{19}$$

Eliminating v_1 and n_1 from the Eqs. (10) and (11), Poisson's equation reduces to

$$\left[1 - \frac{\omega_p^2}{\omega^2}\right]E_1 = 0, \tag{13}$$

where we have introduced the characteristic plasma frequency, ω_p defined by $\omega_p^2 = \frac{n_0e^2}{\epsilon_0m}$.

Clearly, in order to have nontrivial solutions of Eq. (13), one must require that $\omega = \pm\omega_p$, which is the classical result: the frequency of the electrostatic wave has a frequency equal to the plasma frequency. Consider next a situation when the undisturbed situation corresponds to an electron fluid streaming through a neutralizing stationary ion background. In this case the zero order velocity of the electron fluid does not vanish, i.e., $v_0 \neq 0$. Linearizing as before around the background solution, Eqs. (4), (5) and (6) become

$$\frac{\partial v_1}{\partial t} + v_0 \frac{\partial v_1}{\partial x} = -\frac{e}{m}E_1 \tag{14}$$

$$\frac{\partial n_1}{\partial t} + n_0 \frac{\partial v_1}{\partial x} + v_0 \frac{\partial n_1}{\partial x} = 0 \tag{15}$$

and

$$\frac{\partial E_1}{\partial x} = -\frac{e}{\epsilon_0}n_1 \tag{16}$$

respectively.

Eliminating n_1 and v_1 as before and substituting into Poisson's equation, one finds

$$\left[1 - \frac{\omega_p^2}{(\omega - kv_0)^2}\right]E_1 = 0 \tag{20}$$

The corresponding condition for nontrivial solutions becomes

$$\omega = \omega_D \pm \omega_p$$

where we have introduced $\omega_D = kv_0$. This result is obviously expected, the characteristic oscillation frequency of the previous case is now shifted by the Doppler frequency ω_D , due to the streaming electron velocity.

Methodology

The most common and widely used methods of simulating plasmas are Particle In Cell (PIC) methods, which treat plasmas as a system of particles, and the Magneto-Hydro-Dynamic (MHD) methods, which treat the plasmas as fluids. Hybrid models also exist, for instance models that treat some parts of the plasma as a fluid (using MHD methods) but others as particles (using PIC methods). The VSIM simulates plasma by PIC method. In this program the equation of motion for each particle is solved by using four operations which is summarized in Figure 1 below.

The transformed equations now read

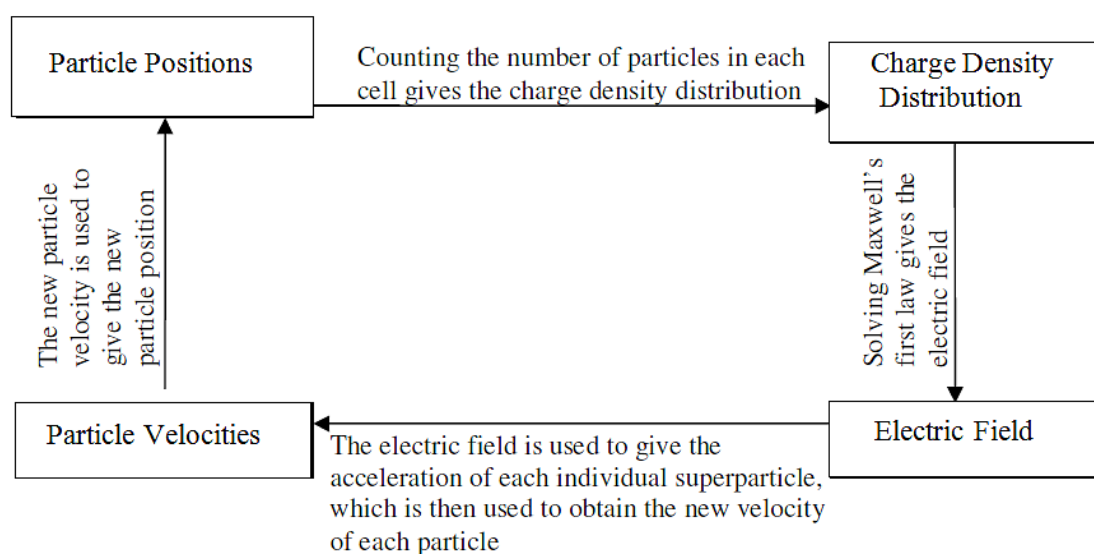


Figure 1. Summary of a computational cycle of the PIC method

Euler’s Method

The simplest of the numerical methods for solving first order Differential Equations (ODE) is the Euler’s method. Euler’s method calculates the slope at a given point, and steps forward a given interval to determine a new point. It then repeats this process at the new point.

The force acting on a particle moving in electric and magnetic field is given the eqn of motion;

$$F = ma = \frac{d^2r}{dt^2} = \frac{q}{m}(E + v \times B) \quad 21$$

Since in this problem, consideration is given only to the electric field component; equation(21) reduces to

$$\frac{d^2r}{dt^2} = \frac{q}{m} E \quad 22$$

Expressing equation(22)in one dimension gives

$$\frac{d^2x}{dt^2} = \frac{q}{m} E . \quad 23$$

This is a well- known ODE defining the acceleration, the second derivative of the position. This can be broken into two differential equations relating the position to its derivative, velocity, and the velocity to its derivative, acceleration. These coupled differential equations are solved simultaneously using Euler’s method. Because the velocity affects the position, the new position is calculated before the new velocity.

$$\frac{dv}{dt} = \frac{q}{m} E = f(t, v) \quad 24$$

$$\frac{dx}{dt} = v = f(t, x) \quad 25$$

Suppose $t_0, t_1, t_2, t_3, \dots$ are successing time values with equal time interval (step) Δt , by separating variables, the differential equation in (24) becomes;

$$dv = \frac{q}{m} E dt \quad 26$$

If $v_0, v_1, v_2, v_3, \dots$ are respective values of the velocities at each time, integrating (26) from t_0 to t_1 with respect to t , at the same time v changes from v_0 to v_1 we get;

$$\int_{v_0}^{v_1} dv = \frac{q}{m} E \int_{t_0}^{t_1} dt \quad 27$$

$$v_1 - v_0 = \frac{q}{m} E (t_1 - t_0) \quad 28$$

But $\Delta t = t_1 - t_0$ therefore eqn. (28) becomes;

$$v_1 = v_0 + \Delta t \frac{q}{m} E_0$$

Similarly, for the range $v_1 \leq v \leq v_2$ we have;

$$\int_{v_1}^{v_2} dv = \frac{q}{m} E \int_{t_1}^{t_2} dt \quad 29$$

$$v_2 = v_1 + \Delta t \frac{q}{m} E_1 \quad 30$$

Proceeding in this way, the general formula is obtained as

$$v_{n+1} = v_n + \Delta t \frac{q}{m} E_n \quad 31$$

In this fashion, one can get the general equation for the position as

$$x_{n+1} = x_n + \Delta t v_n$$

The error in Euler’s method can be reduced, but not eliminated, by choosing a smaller step size, though for significantly greater accuracy it is necessary to make use of a different method (Daniel, 2007).

Leap- Frog Method

Another method that builds on Euler is the Leap Frog Method. This method is commonly used as a replacement to Euler. This method calculates the new position based on the velocity at a given time and updates the velocity based on the next time step i.e. the new position data is calculated by leap-frogging over known velocity data and vice-verse .This offsets the calculation of the position and velocity by a time step giving it its name as the positions and velocities “leap frog” past each other. Leap-frog algorithm based on staggering the time levels of the velocity and position by half time step: $x_p(t = n\Delta t) \equiv x_p^n$ and $v_p(t = (n + \frac{1}{2})\Delta t) \equiv v_p^{n+\frac{1}{2}}$. The advancement of position from time level n to time level $n + 1$ uses the velocity at mid-point, $v_p^{n+\frac{1}{2}}$ and similarly the advancement of the velocity from time level $n - \frac{1}{2}$ to $n + \frac{1}{2}$ uses the mid- point position x_p^n (Daniel,2007), as illustrated in Fig. 2.

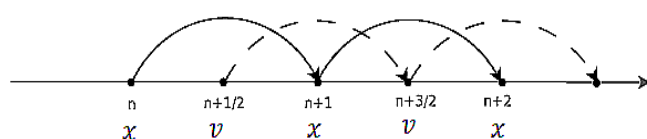


Figure 2: Visual representation of the leap-frog algorithm. Each line represents an operation; when two lines overlap, the line underneath was performed first. Electric field data is dependent on position data.

Among the three methods for time-integration described above the leapfrog scheme was chosen above the other two methods because it is simple and accurate. On the other hand, the Euler’s method which is simple rarely works well in practice and the predictor-corrector method on the other hand is computationally expensive due to recalculation of the electric field.

In this present work, the target is to observe the simulation behaviour of two

Result and discussion

The first column of a Table 1 gives the electron beam velocities at which the simulation was performed, while the second column represents the corresponding kinetic energy of plasma electron.

Fig. 3 (a) demonstrates how the initial phase space from the onset of the simulation shows two counter streaming horizontal beams

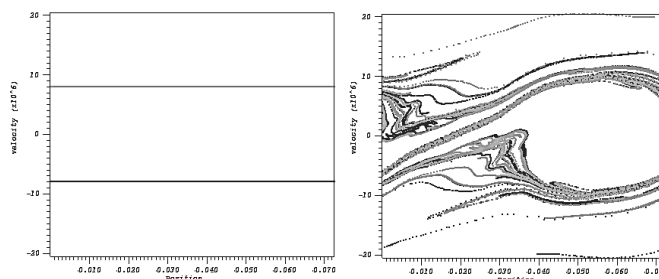


Figure 3: initial and final phase space of counter-streaming electron beams with magnitude(7.89×10^6 m/s).

The two counter-streaming beams interact with each other through the field they generate. The two beams are coupled by a common electric field which is unstable. The two interacting beams transfer their streaming energy to the generated electric field. The unstable electric field grows and then acts on the beams, causing them to mix or bunch and eventually destroys the beams or scatters the particles in phase space. At different value of the streams velocities, the instability tends to increase, causing more turbulent of the beams as the velocity increases. Some electrons are observed travelling with velocity greater than their initial velocity which is as a result of gain in energy from the wave, thus increasing their speed at the expense of the wave

streaming electron beam in terms of kinetic energy of electron as:

- i. The electron beam velocity is varied from a value of about 3.58×10^6 m/s to 7.89×10^6 m/s and
- ii. The particle density is varied from a value of about $1.05 \times 10^{14}m^{-3}$ to $5.84 \times 10^{14}m^{-3}$.

with velocity of 7.89×10^6 m/s, one lying in the positive velocity axis while the other lying in the negative velocity axis. At the end of the interaction, the final phase of the counter streaming electron beams is simulated and depicted in Fig.3.(b).In other words, Figs.3 (a) and (b)illustration only the initial and final phase space of the counter streaming beams, without giving information about the kinetic energy of the electron beams

energy, which consequently decreases wave speed (Anderson *et al.*, 2001).This, can be seen from the phase space plots shown in Fig. 3 (b).

Increasing electron beam Density

Table 2 shows the values of the electron density and its corresponding kinetic energy. As the density of the electrons is increased, the energy of the electrons also increases. Fig 4 (a) is a pictorial representation of one of the visual phase space simulations of the interaction due to particle density, while Fig. 4(b) depicts the resulting kinetic energy of electrons at particle density $1.21 \times 10^{14}m^{-3}$. Previous studies (Macek *et al.*, 2001) predicted that particle

density has lower increasing effect on kinetic energy of plasma than the effect of velocity of plasma electron on the kinetic energy. In this study, it has been observed that increasing the

particle density of plasma electrons increases the kinetic energy of plasma, but the increase in kinetic energy brought about by increasing the velocity of plasma electron is higher.

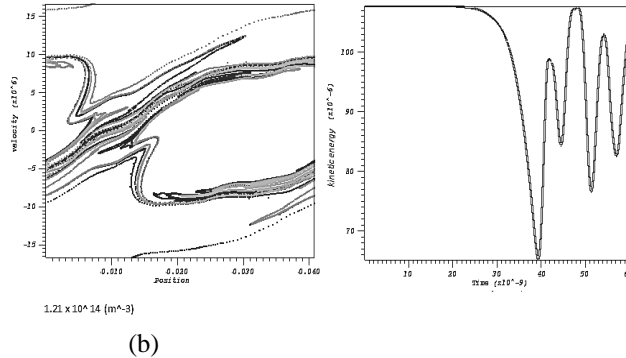


Fig. 4. Phase space simulation of two-stream instability (a) and kinetic energy of electron (b) at particle density of $1.21 \times 10^{14} m^{-3}$

Table 1 Values of kinetic energy resulting From electron beam velocity

Table 2 Values of kinetic energy resulting from electron density

Electron beam velocity (m/s) $\times 10^6$	Kinetic energy of plasma (joule) $\times 10^{-6}$
3.58	19
3.94	25.3
4.37	34.6
4.93	49.6
5.04	53
5.38	64.5
5.65	75
5.73	78
5.86	83.5
6.12	95
6.49	113
6.68	123
6.89	135
6.92	137
7.59	180
7.72	190
7.88	202
7.98	210

Electron Density (m^{-3}) $\times 10^{14}$	Kinetic energy of plasma (joule) $\times 10^{-6}$
1.05	100
1.21	108
1.45	118
1.69	127
1.88	134
2.13	143
2.39	151
2.63	159
2.8	164
2.97	169
3.11	173
3.39	180
3.56	185
3.77	190
3.94	194
4.36	204
4.79	214
4.98	218
5.84	237

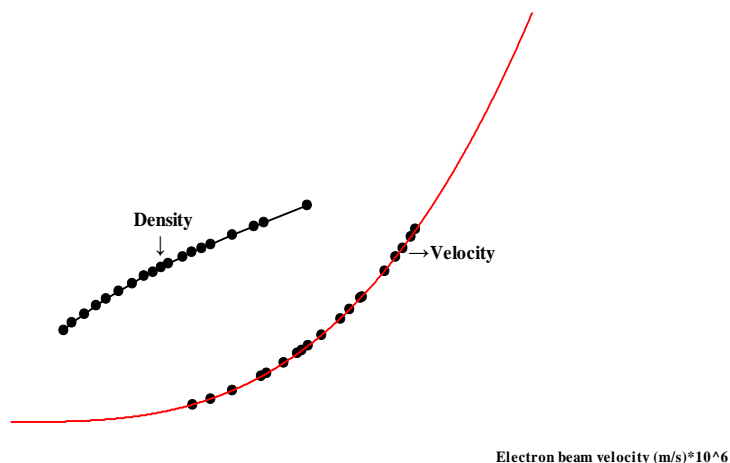


Figure 5: Combined plots of increasing density and velocity of plasma.

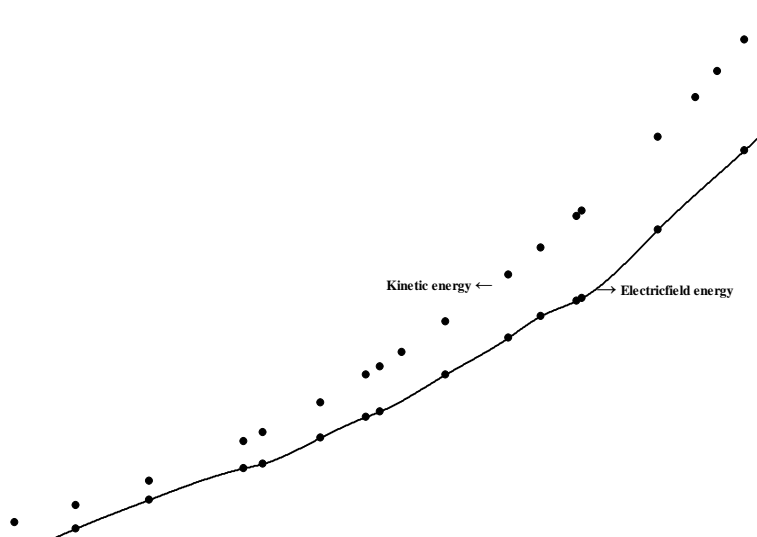


Figure 6: Comparison of kinetic energy and electric field energy of plasma.

As is observed in both Tables 1 and 2, increasing the beam velocity increases the kinetic energy of plasma electron at a higher rate than that caused by increasing the electron beam density. In other words, both increase in electron beam velocity and density lead to increase in the kinetic energy of plasma. However, kinetic energy of plasma is increased more when the electron beam velocity is increased than when the electron beam density is increased. This is clearly shown in Fig.5 where the kinetic energy of plasma electron is plotted against beam velocity as well as particle density. From the graph, it is observed that increase in particle density shows a

more slow increase in kinetic energy of plasma than that of increase in beam velocity. A similar feature was observed by Moroslav in numerical studies of plasma (Moroslav, 2015), and (Horky and Miloch^{a, b}, 2015) in their studies of kinetic plasma instability due to charge exchange and elastic collision, as well as Numerical study on the instability of the weakly collisional plasma in electric and magnetic fields. Therefore, beam velocity increases the plasma kinetic energy in geometrical progression as compared to that of particle density.

The energy generated by the electric field grows exponentially as the energy generated by

the electron beam velocity. However, the electrostatic field energy does not exceed the kinetic energy of the particle (electrons) as it is clearly shown in Fig. 6. This observation was seen in the study of transverse two-stream instability in the presence of strong species-species and image force (Laslett *et al.*, 2008 and Miroslav and Petr, 2013).

Conclusion

When the two beams are propagated against each other they interact via the wave they generate. As the beam velocity is increased the electric field energy also increases. As both velocity and density of the streaming beams are increased, the peak energy of the particles increased in both cases, although at different rates. This is evidence of the fact that throughout the simulation the streaming beams become thermalized, thus making the plasma to become more heated. From the simulation results increase in both particles density and velocity increases the kinetic energy of the particle. However, increase in particle velocity increase the energy more than the increase in particle density does. Also, the electric field energy and energy of the particles grows exponentially and it can be seen that the electrostatic energy does not exceed the energy of the particles.

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