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Optical model analysis of $\alpha + {}^{40}Ca$ E_{lab} = 104 and 141.7 MeV using a mas-dependent M3Y-type effective interaction

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Abstract

Optical model analysis of the elastic scattering analysis of $\alpha + {}^{40}\mathrm{Ca}$ $E_{lab} = 104$ and 141.7 MeV have been performed using the folding model approach of the nuclear reaction video (NRV) knowledge base code. The M3Y-type effective nucleon-nucleon interaction derived using the lowest order constrained variational (LOCV) approach for mass number A = 40 was used to obtain the results. Elastic scattering differential cross section data were obtained for this system at two bombarding energies. The calculated differential cross sections are in good agreement with the experimental results and the anticipated mass dependence of the M3Y-type effective interaction improved the results at forward angles. This shows that, the mass dependence of the M3Y-type effective interaction in our results should be of value in the description of the scattering of other nuclear systems.

Key words: Mass Dependent M3Y-type Interaction, Double Folding Model, Optical Model, Optical Potential, Elastic Scattering, Differential Cross-Sections.

Introduction

The studies of α-particles scattering from light heavy -ion (HI) nuclei have two distinctive features. The first one known as anomalous large angle scattering (ALAS), observed in the elastic scattering of α particle from several targets as well as in non-elastic processes. The second one is the nuclear rainbow scattering observed at higher energies which is characterized by a sharp decrease of the cross section beyond a certain scattering angle known as the grazing angle (Hassanain, 2011). These problems have in the past been approached from diverse directions using different local potentials. In nuclear reaction processes the nucleus-nucleus potential is one of the most important quantities, for example in elastic scattering of α-particle and light heavy-ion (HI) systems (Gao-Long, Hao, & Xiao-Yun, 2009).

The HI systems are best understood in terms of empirical parameterization of the nuclear potential. Hence, it is desirable to relate the nucleus-nucleus nuclear interaction to the nucleon-nucleon nuclear interaction. The nuclear potential may be obtained by integrating a nucleon-nucleon interaction over the matter distributions of the two colliding nuclei. This approach is called the folding model. It produces the dominant part of the real optical potential for nuclear reactions (elastic scattering, fusion reaction). The real part of the nuclear potential obtained by folding in the density distribution function of two interacting nuclei with the density-dependent Michigan three Yukawa (M3Y) effective interaction (Vina, Lozano, & Madurga, 1981) has shown to provide good and usually a phenomenological Woods-Saxon form is used for the imaginary parts of the optical potential (Majka & Srokowski, 1978). Most part of the experimental data for various nucleus-nucleus reactions have been analysed using the Woods-Saxon type and double-folding potentials. But only few reactions and potential properties have been considered in elastic scattering reactions of α-particle and systems of light heavy ions. So, it is important to analyse this system employing the double folding model to show the mass-dependent effects of the M3Y-type interaction in the ALAS feature of α-particle scattering from light heavy ions by determining the elastic scattering cross sections of $\alpha + {}^{40}\text{Ca}$ at $E_{lab} = 104$ and 141.7 MeV.

In our present work, we use an effective nucleon-nucleon interaction fitted from the lowest order constrained variational approach within the double folding model approach to provide a unified description of the optical model analysis of $\alpha + {}^{40}\text{Ca}$ at $E_{lab} = 104$ and 141.7 MeV. The nucleus-nucleus potential is a key ingredient in the analysis of nuclear reactions. By using the potential between nuclei we can evaluate the cross sections of different nuclear reactions (Denisov & Davidovskaya, 2010).

This paper is organized as follows: In Section 2, a brief summary of an expression of the mass-dependent M3Y -type effective interaction for $A = 40$ system is given. In Section 3, we define the folding model as included in the nuclear reaction video knowledge base code. In Section 4, we present the results of our findings and other relevant discussions are made. Finally, we make the conclusions of our findings in Section 5.

The Effective Nucleon-Nucleon Interaction

The effective interaction applied to the folding model in this work is the M3Y-type effective interaction constructed from the lowest order constrained variational (LOCV) approach for the A = 40 nuclear system. To construct this effective nucleon -nucleon interaction, the central part of the interaction V_{pt} is written in the form (Brandan & Satchler, 1997).

$$V_{pt} = V_{00}(r_{pt}) + V_{01}(r_{pt})\tau_p \cdot \tau_T + V_{10}(r_{pt})\sigma_p \cdot \sigma_T + V_{11}(r_{pt})\sigma_p \cdot \sigma_T\tau_p \cdot \tau_T. \quad (1)$$

where σ, τ are the Pauli matrices for spin and isospin respectively.

Frequently, instead of equation (1), the nucleon-nucleon interaction is expressed in terms of the total spin (singlet S or triplet T) of the two nucleon system and the parity of its relative orbital angular momentum. In these terms the spin-isospin independent component V_{00} becomes (Bertsch, Borysowiicz, McManus, & Love, 1977; Brandan & Satchler, 1997)

$$V_{00} = \frac{1}{16}(3V_{SE} + 3V_{TE} + V_{SO} + 9V_{TO}). \quad (2)$$

where SE, TE, SO and TO are the singlet - even, triplet - even, single - odd and triplet - odd channels respectively.

This representation is particularly convenient when considering exchange, for the effect of the exchange operator is to change the sign of the odd-state components and leave the even-state ones unchanged. Thus the interaction $\hat{V}_{00}$ appropriate for the knock-on exchange term becomes

$$\hat{V}_{00} = \frac{1}{16}(3V_{SE} + 3V_{TE} - V_{SO} - 9V_{TO}). \quad (3)$$

This change of sign also implies some cancellation of the odd-state contribution when the direct and exchange potentials are added, making the result less sensitive to the choice of odd state interaction.

To determine this effective nucleon - nucleon interaction which is suitable for the present calculation, we select the data of Table VII from the determined best fit interaction strengths (in MeV) for $A = 40$ from Ref. (Fiase, Devan, & Hosaka, 2002). We obtain the effective interaction for A = 40 as,

$$V_{00(D)}(r) = 11012\frac{e^{-4r}}{4r} - 2359\frac{e^{-2.5r}}{2.5r}, \quad (4)$$

and

$$\hat{V}_{00?EX?}(r) = 1039.25\frac{e^{-4r}}{4r} - 1503.94\frac{e^{-2.5r}}{2.5r} - 7.847\frac{e^{-0.7072r}}{0.7072r}, \quad (5)$$

where $V_{00(D)}$ and $\hat{V}_{00(EX)}$ are the direct and exchange terms respectively.

Density -Dependent Effective Nucleon - Nucleon Interaction

It has long been recognized that the effective interaction between two nucleons in a nucleus depends upon the density of the surrounding medium. Indeed, this density is required for nuclear matter to saturate rather than collapse. Saturation requires that the attraction weakens as the density increases. In the present calculation, the density-dependent effective interaction is taken in the form (Moharram & El-shal, 2002)

$$V_{D(EX)}(\rho, E, r) = f(\rho)\, V_{D(EX)}(r), \quad (6)$$

where $V_{D(EX)}$ are the original effective nucleon-nucleon interactions.

Concerning the explicit form of $f(\rho)$ the one we take under consideration is the form of the density dependence of the M3Y effective interaction, DDM3Y reported in Ref. (Khoa & von Oertzen, 1993). This has the form,

$$f(\rho) = C(1 + \alpha \exp(-\beta\rho)), \quad (7)$$

where C, α and β are parameters.

This density dependence changes the sign of the interaction at high densities which is of crucial importance in fulfilling the saturation condition for the nuclear matter equation of state. The parameters C, α and β are fitted in order to ensure the saturation of nuclear matter and to reproduce the correct value of the nuclear matter binding energy $\in = -16$ MeV at $\rho = \rho_0 \approx 0.17$ fm^{-3}. The parameters of the function $f(\rho)$ of our M3Y -type effective interaction for A=40 are listed in Table 1.

It should be noted that while the original M3Y effective interaction is mass - independent, several calculations based on it have indicated some mass dependence in the form (Khoa & von Oertzen, 1993)

$$V_{D(EX)}(\rho, E, r) = f(\rho)\left(1 - \frac{0{,}002E}{A}\right)V_{D(EX)}(r) \quad (8)$$

where A is the mass of the target nucleus and E is the energy of the projectile. In our earlier calculations (Fiase *et al.*, 2002), the mass dependence of our M3Y - type effective interaction was systematically investigated for the A = 16, A = 40 and A = 90 nuclear systems, In the present calculations we have used the effective interaction appropriate for the A = 40 nuclear system.

Table 1: Parameters of the density dependent M3Y-type interaction

Interaction	C	A	B
M3Y-type (A = 40)	0.4089	2.2923	2.9605

The Knock-On Exchange

The only effect of antisymmetrization under exchange of nucleons between the two nuclei that are normally included in the folding model is the single nucleon knock -on exchange in which the two nucleons that are interacting via V_{pt} are interchanged as in Eqn. (3). At least two groups (Love & Owen, 1975) calculated this knock-on exchange potential and concluded that it could be estimated quite accurately by adding a zero -range pseudo -potential to the interaction V_{pt}, namely, by replacing V_{pt} by

$$V_{pt}' = V_{pt}(1 - P_{pt}) \rightarrow V_{pt} + J(E)\delta r_{pt}. \quad (9)$$

This form of effective nucleon -nucleon interaction has been used in calculations of the exchange potential for alpha-particle-nucleus and heavy-ion scattering in which the finite range was accounted for explicitly (Brandan & Satchler, 1997). The magnitude of $J(E)$ has been determined empirically (Gupta, 2010) by comparing cross sections for protons scattering from various targets and at various energies up to 80 MeV. In the present calculation, the magnitude of $J(E)$ for our M3Y -type effective interactions is estimated by comparing the cross sections for alpha -particle and light heavy ion scattering within a wide energy range and the result is expressed in the form

$$J(E) \sim -270\left[1 - 0.005\left(\frac{E}{A}\right)\right] \text{MeVfm}^3 \quad (10)$$

Folding Potential

The double-folding procedure is applied to calculation of heavy -ion interaction potential $V_F(r)$ using realistic nucleon -nucleon (NN) interaction. In this model the optical potential for heavy-ion scattering is obtained by averaging an appropriate NN interaction over the matter distributions within the two colliding ions (Satchler & Love, 1979; Zagrebaev & Kozhin, 1999) as

$$V_F(r) = \iint \rho_1(r_1)\rho_2(r_2)V_{NN}(r - r_1 + r_2)d^3r_1d^3r_2 \quad (11)$$

where the integration is performed over the projectile and target volumes, V_{NN} is the effective nucleon-nucleon interaction and $\rho_i(r_i)$ are the density distributions of nuclear matter in the nuclei (i = 1, 2). The nuclear and charge densities of the projectile or target nuclei are described using either the Fermi -type function or the Gaussian-type function (Zagrebaev & Kozhin, 1999).

Results and Discussion

The elastic scattering data of $\alpha + {}^{40}Ca$ at $E_{lab} = 104$ and 141.7 MeV was obtained using the density -dependent form of the M3Y-type effective interaction derived using the LOCV approach and the results are compared to experimental data and those in Refs.(Abdullah et al., 2005; Brandan & Satchler, 1997; G ill et al., 1979). The real folded potential $V_F(r)$ was constructed using our M3Y –type effective interaction of equations (4) and (5) together with the zero -range exchange term of equation (9). The parameter values for optimum fits are listed in Table 2.

Table 2: Parameters of elastic scattering of $\alpha + {}^{40}Ca$ at $E_{lab} = 104$ and 141.7 MeV

System	interaction	E_{lab} (MeV)	N_R	N_I	σ_r(mb)	σ_{tot}(mb)
$\alpha + {}^{40}Ca$	M3Y-type A=40	104.00	1.20	0.20	774.92	1854.69
		141.70	1.58	0.20	745.98	1941.11

The potential $V_F(r)$ was fed into the folding model search code of NRV. It was then multiplied with the density dependent factor of equation (7) and by the renormalization factors N_R and N_I for the real and imaginary potentials respectively. The renormalization factors were varied in order to optimize the fit to the data as shown in Table 2. Table 2 summarizes the results given the optimum values of the differential and total cross sections, renormalization factors N_R and N_I and the magnitude of the real folded potential as well as the values of other parameters needed for the analysis.

The real folded potentials were obtained with a renormalisation factor of 1.2 and 1.58. The magnitudes of the potential were also found to be approximately $200\ MeV$ and $260\ MeV$ deep at $r = 0$ while the empirical ones are $\approx 126\ MeV$ (Gill et al., 1979). The imaginary parts of the optical potential were also obtained with a renormalisation factor of 0.2. The magnitude of the potential for the imaginary part were obtained to be approximately 35 MeV at $E_{lab} = 104\ MeV$ and $E_{lab} = 141.7\ MeV$ respectively.

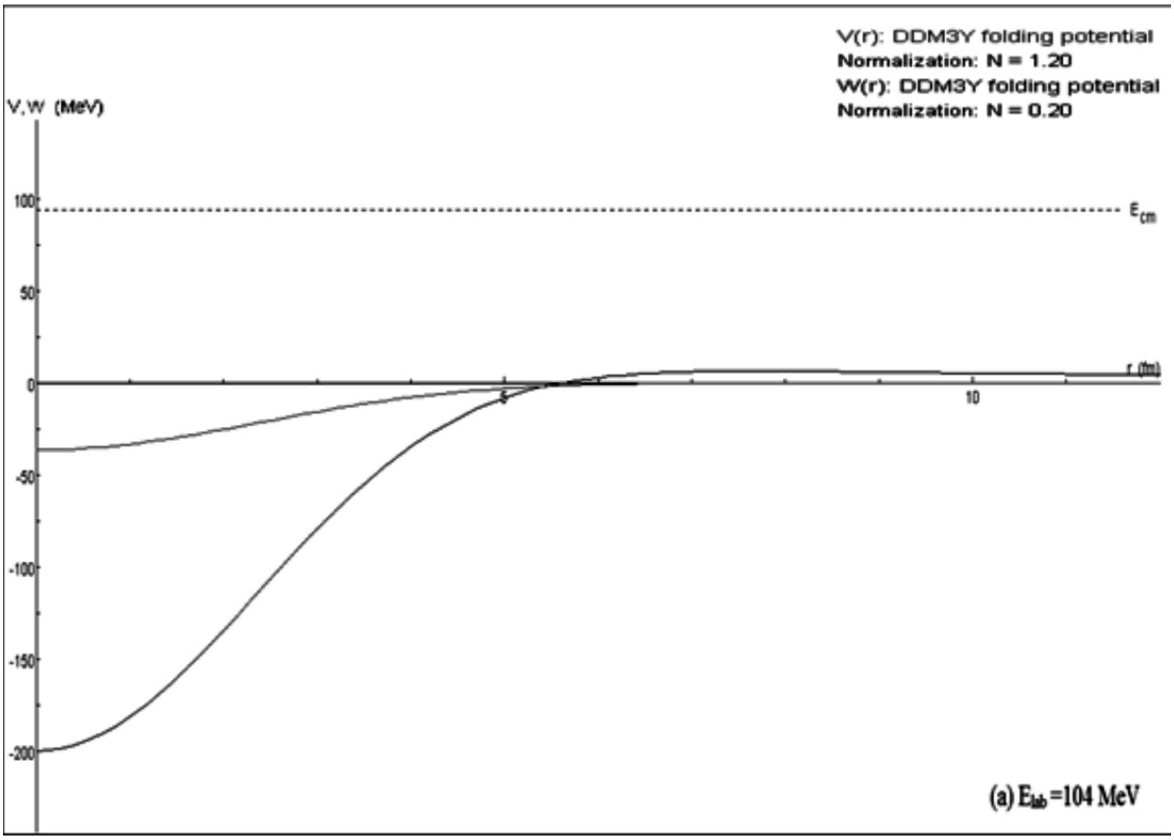

Figure 1: Radial shape of the folded potential of elastic scattering of $\alpha + {}^{40}Ca$ at $E_{lab} = 104\ MeV$

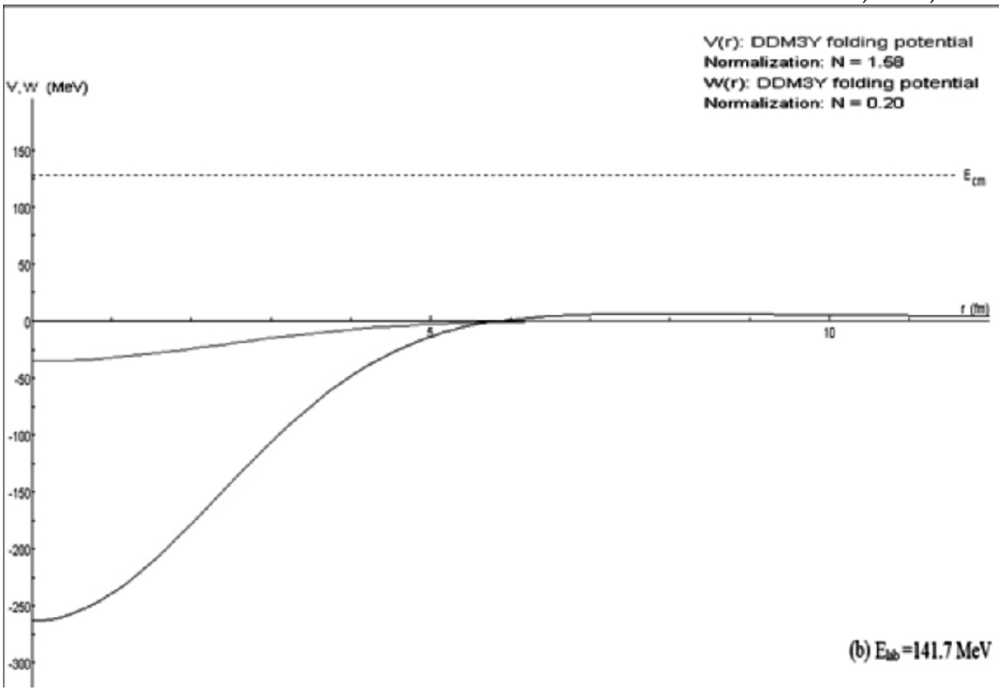

Figure 2: Radial shape of the folded potential of elastic scattering of $\alpha + {}^{40}Ca$, at $E_{lab} = 141.7\ MeV$

The plot of the elastic scattering angular distributions of $\alpha + {}^{40}Ca$ at $E_{lab} = 104\ MeV$ and $141.7\ MeV$ using M3Y-type interactions were also obtained and compared to the experimental data. The results are displayed in Figures 3 and 4. The solid lines of Figures 3 and 4 represent the theoretical result and the dotted lines are for the experimental data.

Following the data displayed in Figure 3, it can be seen that the calculated results show some good agreement with the experimental data. This interaction has shown a remarkable improvement in fitting the experimental data at angles greater than 80° which has always been a challenge as reported in (Abdullah et al., 2005; Khoa & von Oertzen, 1995; Satchler & Love, 1979; Vina et al., 1981). From these sets of data, it is quite reasonable to agree that, the M3Y-type interaction fit the experimental data very well at large angles up to 140°. The ability of this interaction to fit the experimental data at large angles ($> 100°$) is a clear indication that the presence of the mass dependence in our effective interaction is necessary to improve the fit.

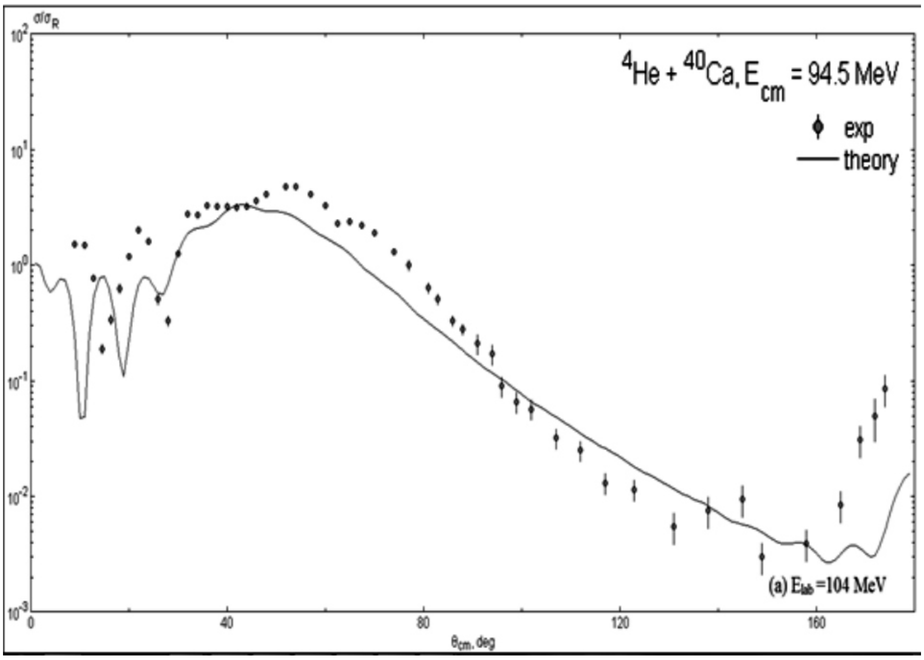

Figure 3: Elastic scattering angular distributions of $\alpha + {}^{40}Ca$ at $E_{lab} = 104\ MeV$

The analysis of the theoretical results has shown some diffraction patterns which are in good agreement with the experimental results. These diffraction patterns are characterized by heavy minimas and maximas at small angles between 10 ° and 40°. Refractive features are also observed at angles greater than 120°. This refractive feature of the α particle scattering has further ascertained and affirmed the M3Y -type mass - dependent interaction as a good interaction in the de scription of α -particle and light heavy ion scattering and it can be seen as a correction to the ALAS feature reported in (Abdullah et al., 2005; Brandan & Satchler, 1997; Khoa & von Oertzen, 1995; Satchler & Love, 1979).

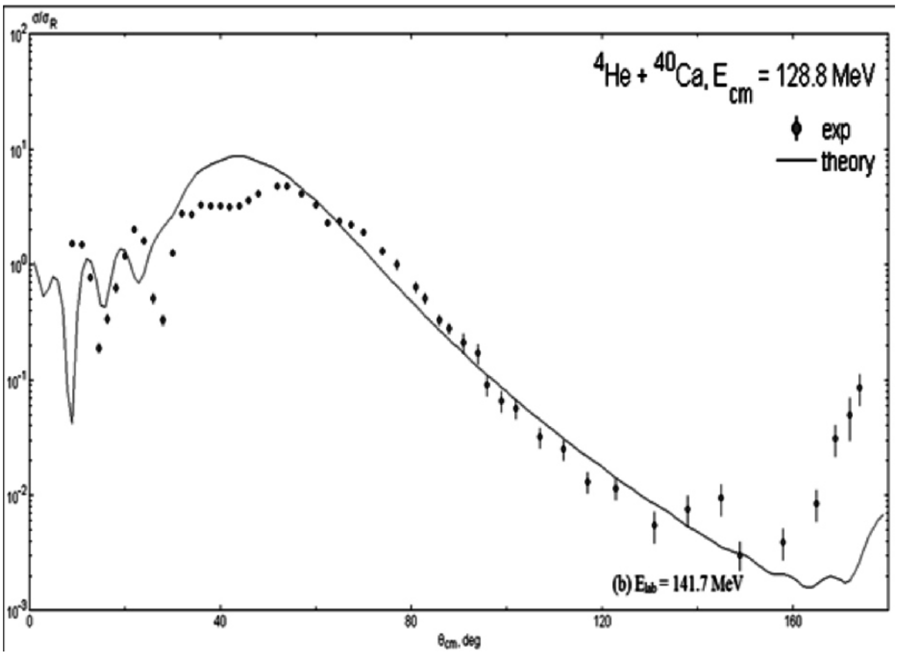

Figure 4: Elastic scattering angular distributions of $\alpha + {}^{40}Ca$ at $E_{lab} = 141.7$ MeV

Finally we have seen that, the application of the M3Y-type effective interaction to the scattering of alpha -particle produced similar results as discussed in (Gill et al., 1979; Khoa & von Oertzen, 1995) and the agreement of the data at large ($> 100°$) scattering angles has established the mass - dependent density dependent M3Y- type interaction as a physically correct interaction.

Conclusion

The elastic scattering cross sections of $\alpha + {}^{40}Ca$ at $E_{lab} = 104$ MeV and $E_{lab} = 141.7$ MeV using our M3Y-type effective interaction are computed with a folded potential of Section 3 in the NRV search code. The search was conducted and results are presented in Tables 2. The results are also disp layed for the depth of the real potentials and the differential cross sections as in Figures 1 -4. The values of N_R from Table 2 show that, the renormalization values are slightly larger than unity. This indicates that on the average the folding model with our M3Y-type effective interaction predicts the alpha-particle real potential quite accurately. Indeed even the deviation of the mean value of N_R from unity could be due to the underestimation of the exchange contribution in using the zero range pseudo potential approximation. It can be seen from Figures 3 and 4 that, good approximations of elastic scattering angular distributions using our M3Y-type effective interactions at angles above 100 ° has established the interaction as a good interaction for folding model studies and has shown an improvement in the anomalous large angle scattering reported (Abdullah et al., 2005; Brandan & Satchler, 1997; Gill et al., 1979; Khoa & von Oertzen, 1995) . This agreement is satisfactory, particularly since no free parameter is used in the construction of the real potential (except as N_R is different from unity) and it should be noted that a bad real potential cannot be healed by an imaginary partner.

Conclusively, this analysis has shown that, the mass dependence of our M3Y -type effective interaction has improved the results of the α -particle scattering as seen in the analysis of $\alpha + {}^{40}\mathrm{Ca}$ at $E_{lab} = 104$ and 141.7 MeV by fitting the experimental data at angles greater than 80°. The present calculation with our effective interaction has confirmed the appropriateness of our M3Y − type effective interaction in recent calculations (Ochala & Fiase, 2018).

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