Modeling Fire Spread in a Real-Time Coupled Atmospheric-Vegetation Fire: An Analytical Approach

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Abstract
The ability to analyse the rate of fire spread outbreak in a real-time coupled Atmospheric-vegetation fire has become increasingly vital as forest fire fighters are building diverse kinds of models to combat the dangers/effects of fire spread across a given fire vicinity. This paper theoretically examines the analysis of fire spread in a real fire environment. A partial differential equations (PDE) governing the phenomenon is presented. The analytical solution of the model is obtained via direct integration and eigenfunction expansion technique, which displays the influence of the parameters involved in the system. The effect of change in parameters such as Frank-Kamenetskii number, Radiation number, Peclet energy number and Activation energy number are presented graphically and discussed. The results obtained show that Frank-Kamenetskii number, Radiation number, Peclet energy number, and Activation energy number all reduced transient state temperature.

Keywords & Phrases: Combustion, crown fires, fire spread, forest fire, ground fire, spotting fire, temperature, vegetation fire.

Introduction

Interest in mathematical models of vegetation fire is caused by a large number of scientific difficulties. Goldammer and Ronde (2004) noted that forest fire models have been developed since 1940 to the present, but a lot of chemical and thermodynamic questions related to fire behaviour are still to be resolved. Forest fires are divided into underground (peatbog) fires, surface fires, active crown fires, running crown fires (also called independent crown fires), and mass fires. A number of researchers notice, that running crown fires possess the greatest speed of propagation. They are extremely dangerous and very difficult to fight, thus mathematical modelling is represented as an important problem (Pastor *et al.,* 2003).

Fire models can be classified by the type of fire in consideration. There are four major types of fire, and each one has different behaviour. Therefore, physical systems and equations vary from one to another. Pastor *et al.* (2003) analysed an extensive list of forest fire models and classified them according to their typology. Basically, the four types of fire models are: surface fire, crown fire, spotting fire and ground fire.

Surface fire models deals with the fire that burns the vegetation closed to the surface, such as brush, small trees, or herbaceous plants. Crown fire models are somewhat complementary to surface fire models and studied how the fire spreads over the canopy of trees in a given forest. Spotting fire models provides the equations to analyse those new fire caused by incandescent pieces of the main fire transported out of the main fire perimeter. Finally, ground fire models focused their attention on those physical processes that occur in the substrate of the soil when a fire takes place (Carlos, 2014)

Heat, an important aspect of the fire triangle, is the energy transferred between an object of greater temperature to an object of lower temperature. It is this heat energy that is crucial in beginning the evaporative or preheating phase of combustion (Johnson and Miyanishi, 2001). Temperature determines the ease of combustion of wildland fuels. Therefore, higher temperatures heat forest fuels and predispose them to ignition provided that an adequate ignition source becomes readily available (lightning or some anthropogenic source).

Vegetation fire and other natural hazards are cases widely studied by scientific community due to its huge environmental, social and economic impact that they produce virtually every year around the world. Considering forest fire, several preventive actions can be applied to reduce the forest fire risk/spread, but unfortunately, they do not wholly eliminate/eradicate the diverse factors unleashed by a fire. Due to this, our effort is targeted on mitigating the escalation of vegetation fire by showcasing the effects of some parameters such as Frank-Kamenetskii number, Radiation number, Peclet energy number, and Activation energy number on the transient state temperature at a fire scene. This will be achieved via direct integration and eigenfunction expansion technique.

Model Formulations

Following Perminov (2018), a wildfire model is formulated based on balance equations for energy and fuel, where the fuel loss due to burning corresponds to the fuel reaction rate. The respective equations governing forest fires propagation are:

Volume fraction of dry organic substance

$$\frac{\partial \varphi_s}{\partial t} = -k_1 \varphi_s e^{-\frac{E_1}{RT}} \quad (1)$$

Volume fraction of moisture

$$\frac{\partial \varphi_m}{\partial t} = -k_2 \varphi_m T^{\frac{1}{2}} e^{-\frac{E_2}{RT}} \quad (2)$$

Volume fraction of coke

$$\rho_c \frac{\partial \varphi_c}{\partial t} = \alpha_c k_1 \rho_s \varphi_s e^{-\frac{E_1}{RT}} - \frac{M_c}{M_1} k_3 S_\sigma \rho_g \varphi_c C_{ox} e^{-\frac{E_3}{RT}} \quad (3)$$

Mass concentration of oxygen

$$\left.\begin{aligned}&\rho_g\left(\frac{\partial C_{ox}}{\partial t}+v\frac{\partial C_{ox}}{\partial x}\right)=\frac{\partial}{\partial x}\left(\rho_g D_T\frac{\partial C_{ox}}{\partial x}\right)-\frac{\alpha}{C_{pg}\Delta h}\left(C_{ox}-C_{ox_\infty}\right)-\\&(1-\alpha_c)k_1\rho_s\varphi_s C_{ox}e^{-\frac{E_1}{RT}}-k_2\rho_m T^{\frac{1}{2}}\varphi_m C_{ox}e^{\frac{-E_2}{RT}}-k_3S_\sigma\rho_g\left(1+\frac{M_c}{M_1}C_{ox}\right)\varphi_c C_{ox}e^{\frac{-E_3}{RT}}\end{aligned}\right\}\quad(4)$$

Energy balance equation

$$\left.\begin{aligned}&\left(\phi\rho_g C_{pg}+(1-\phi)\sum_{i=1}^{s+m+c}\rho_i C_{pi}\varphi_i\right)\frac{\partial T}{\partial t}+\rho_g C_{pg}v\frac{\partial T}{\partial x}=\frac{\partial}{\partial x}\left(\lambda_T\frac{\partial T}{\partial x}\right)-\frac{\alpha}{\Delta h}(T-T_\infty)\\&-4K_R\sigma T^4-k_2\rho_m q_2T^{\frac{1}{2}}\varphi_m e^{\frac{-E_2}{RT}}+k_3S_\sigma\rho_g q_3\varphi_c C_{ox}e^{\frac{-E_3}{RT}}\end{aligned}\right\}\quad(5)$$

with initial and boundary conditions:

$$\left.\begin{aligned}&\varphi_s(x,0)=\varphi_{so},\ \varphi_m(x,0)=\varphi_{mo},\ \varphi_c(x,0)=\varphi_{co},\ C_{ox}(x,0)=C_{ox_o},\ C_{ox}(0,t)=C_{ox_\infty}\\&C_{ox}(\mathrm{L},t)=C_{ox_\infty},\ T(x,0)=T_o,\ T(0,t)=T_\infty,\ T(\mathrm{L},t)=T_\infty\end{aligned}\right\}\quad(6)$$

where;

φ_s is the volume fraction of dry organic substance, φ_m is the volume fraction of moisture, φ_c is the volume fraction of coke, C_{ox} is the concentration of oxygen, T is the temperature (in Kelvin), t is the time, x is a coordinate in the system of coordinates connected with the centre of an initial fire (distance), T_∞ is the unperturbed ambient temperature, $k_j, j=1,2,3$ are the pre-exponential factors of chemical reactions, $E_j, j=1,2,3$ are the activation energy of chemical reactions, C is the concentration, R is the universal gas constant, S_σ is the specific surface of the condensed product of pyrolysis (coke), v is the equilibrium wind velocity vector, U is the reference velocity, λ_T is the turbulent thermal conductivity, C_{ox_∞} is the unperturbed density of concentration of oxygen, $P_i, i=(s,m,c)$ is the i^{-th} phase density, that is ρ_s is the density of dry organic substance, ρ_m is the density of moisture, ρ_c is the density of coke, ρ_g is the density of gas phase (a mix of gases), Δh is the crown height, M_c is the molecular mass of carbon, M_1 is the mass of combustible forest material (CFM), C_{pg} is the thermal capacity of a gas phase, $q_j, j=2,3$ defines heat effects of processes of evaporation of burning, D_T is the diffusion coefficient, α is the coefficient of heat exchange between the atmosphere and a forest canopy, α_c is the coke number of combustible forest material (CFM), K_R is the Stefan-BoltzMann constant, $C_{p_i}, i=(s,m,c)$ is the i^{-th} phase of thermal capacity, s is the dry organic substance, m is the moisture, c is the coke, ox is the oxygen (O_2).

Method of Solution

Non-dimensionalisation

Here equation (1) – (6) are non-dimensionalize using the following dimensionless variables:

$$\left.\begin{aligned}&x'=\frac{x}{L},\ t'=\frac{Ut}{L},\ v'=\frac{v}{U},\ \psi_1=\frac{\varphi_s}{\varphi_{so}},\ \psi_2=\frac{\varphi_m}{\varphi_{mo}},\ \psi_3=\frac{\varphi_c}{\varphi_{co}},\ \phi=\frac{C_{ox}-C_{ox_\infty}}{C_{ox_o}-C_{ox_\infty}}\\&\in=\frac{RT_0}{E},\ \theta=\frac{E(T-T_o)}{RT_o^2},\ f=\frac{E_1}{E_3},\ r=\frac{E_2}{E_3}\end{aligned}\right\}\quad(7)$$

and we obtain;

$$\left.\begin{aligned}&\frac{\partial \psi_1}{\partial t} = -a\psi_1 e^{\frac{f\theta}{1+\in\theta}}\\ &\psi_1(x,0)=1\end{aligned}\right\} \tag{8}$$

$$\left.\begin{aligned}&\frac{\partial \psi_2}{\partial t} = -b\psi_2 (1+\in\theta)^{\frac{1}{2}} e^{\frac{r\theta}{1+\in\theta}}\\ &\psi_2(x,0)=1\end{aligned}\right\} \tag{9}$$

$$\left.\begin{aligned}&\frac{\partial \psi_3}{\partial t} = \beta\psi_1 e^{\frac{f\theta}{1+\in\theta}} - \gamma(\phi+q)\psi_3 e^{\frac{\theta}{1+\in\theta}}\\ &\psi_3(x,0)=1\end{aligned}\right\} \tag{10}$$

$$\left.\begin{aligned}&\frac{\partial \phi}{\partial t} + v\frac{\partial \phi}{\partial x} = \frac{\partial}{\partial x}\left(D_1 \frac{\partial \phi}{\partial x}\right) - \beta_1\phi - \beta_2\psi_1(\phi+q)e^{\frac{f\theta}{1+\in\theta}}\\ &-\beta_3(1+\in\theta)^{\frac{1}{2}}\psi_2(\phi+q)e^{\frac{r\theta}{1+\in\theta}} - \beta_4\psi_3(\phi+p)(\phi+q)e^{\frac{\theta}{1+\in\theta}}\\ &\phi(x,0)=1,\ \ \phi(0,t)=0,\ \ \phi(1,t)=0\end{aligned}\right\} \tag{11}$$

$$\left.\begin{aligned}&\frac{\partial \theta}{\partial t} + v\frac{\partial \theta}{\partial x} = \frac{\partial}{\partial x}\left(\lambda_1 \frac{\partial \theta}{\partial x}\right) - \alpha_1(\theta+\gamma_1) - R_a(1+4\in\theta) - \delta\psi_2(1+\in\theta)^{\frac{1}{2}} e^{\frac{r\theta}{1+\in\theta}}\\ &+\delta_1\psi_3(\phi+q)e^{\frac{\theta}{1+\in\theta}}\\ &\theta(x,0)=0,\ \ \theta(0,t)=\sigma_1,\ \ \theta(1,t)=\sigma_1\end{aligned}\right\} \tag{12}$$

where;

$$\left.\begin{aligned}&a=\frac{k_1 L e^{\frac{-fE_1}{RT_o}}}{U},\quad b=\frac{k_2 T_o^{\frac{1}{2}} L e^{\frac{-rE_3}{RT_0}}}{U},\quad \beta=\frac{\alpha_c k_1 \rho_s \varphi_{so} L e^{\frac{-fE_3}{RT_o}}}{U\rho_c\varphi_{co}},\ \gamma=\frac{M_c k_3 S_\sigma \rho_g L}{M_1 U \rho_c}\left(C_{ox_o}-C_{ox_\infty}\right)e^{\frac{-E_3}{RT_o}},\\ &q=\frac{C_{ox_\infty}}{C_{ox_o}-C_{ox_\infty}},\ \ D_1=\frac{D_T}{LU}=\frac{1}{P_{em}},\ \beta_1=\frac{\alpha L}{C_{pg}\Delta h U},\ \beta_2=\frac{(1-\alpha_c)k_1\rho_s\varphi_{so}L}{\rho_g U}e^{\frac{-fE_3}{RT_o}},\\ &\beta_3=\frac{k_2\rho_m T_o^{\frac{1}{2}}\varphi_{mo} L e^{\frac{-rE_3}{RT_o}}}{\rho_g U},\ \beta_4=\frac{k_3 S_\sigma \rho_g \frac{M_c}{M_1}\left[C_{ox_o}-C_{ox_\infty}\right]L\varphi_{co}e^{\frac{-E_3}{RT_o}}}{\rho_g U},\ p=\frac{\frac{M_1}{M_c}+C_{ox_\infty}}{C_{ox_o}-C_{ox_\infty}},\\ &\lambda_1=\frac{\lambda_T}{L\rho_g C_{pg} U}=\frac{1}{P_e},\ \ \alpha_1=\frac{\alpha L}{\rho_g C_{pg} U},\ R_a=\frac{4K_R\sigma L T_o^{3}}{\rho_g C_{pg}\in U},\ \delta=\frac{k_2\rho_m q_2 T_o^{\frac{1}{2}} L\varphi_{mo} e^{\frac{-rE_3}{RT_o}}}{\rho_g C_{pg}\in T_o U},\\ &\delta_1=\frac{k_3 S_\sigma \rho_g q_3 \varphi_{co} L\left(C_{ox_o}-C_{ox_\infty}\right)}{\rho_g C_{pg}\in T_o U}e^{\frac{-E_3}{RT_o}},\ \gamma_1=\frac{T_o-T_\infty}{\in T_o},\ \ \sigma_1=\frac{T_\infty-T_o}{\in T_o}\end{aligned}\right\} \tag{13}$$

Solution of the Model

Using perturbation method, direct integration and eigenfunction expansion technique, the analytical solution of equations (8)—(12) are obtained as follows:

$$\psi_1(x,t)=1+v\left(-A_3\sum_{n=1}^{\infty}A_2\sin n\pi x - a_6\left(t+\left(\begin{aligned}&f(e-2).\\ &\left(\sigma_1 t+\sum_{n=1}^{\infty}\left(A_1 t-\left(\frac{b_n-A_1}{c_2}\right)e^{-c_2 t}\right)\sin n\pi x\right)\end{aligned}\right)\right)\right) \tag{14}$$

$$\psi_2(x,t)=1+v\left(-a_7\left(\begin{array}{l} t+r(e-2)\left(\sigma_1 t+\sum_{n=1}^{\infty}\left(A_1 t-\left(\frac{b_n-A_1}{c_2}\right)e^{-c_2 t}\right)\sin n\pi x\right)+ \\ \frac{1}{2}\in\left(\sigma_1 t+\sum_{n=1}^{\infty}\left(A_1 t-\left(\frac{b_n-A_1}{c_2}\right)e^{-c_2 t}\right)\sin n\pi x\right)+ \\ \frac{1}{2}\in\left(r(e-2)\right)\left(\begin{array}{l} \sigma^2{}_1 t+2\sigma_1\left(\sum_{n=1}^{\infty}\left(A_1 t-\left(\frac{b_n-A_1}{c_2}\right)e^{-c_2 t}\right)\sin n\pi x\right) \\ +\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\left(\begin{array}{l} A_1{}^2 t-\frac{2A_1}{c_2}\left(b_n-A_1\right)e^{-c_2 t} \\ -\frac{\left(b_n-A_1\right)^2}{2c_2}e^{-2c_2 t}\end{array}\right)\sin^2 n\pi x\end{array}\right)\end{array}\right) \\ -a_7\left(A_7+\sum_{n=1}^{\infty}B_1\sin n\pi x\right)\sum_{n=1}^{\infty}A_2\sin n\pi x\right) \tag{15}$$

$$\psi_3(x,t)=1+v\left(\begin{array}{l} a_9\left(A_8 t+A_9\sum_{n=1}^{\infty}\left(A_1 t-A_2 e^{-c_2 t}\right)\sin n\pi x\right) \\ +a_8\left(\begin{array}{l} A_{10}\sum_{n=1}^{\infty}\frac{A}{c_1}e^{-c_1 t}\sin n\pi x+A_{11}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\left(B_2 e^{-c_1 t}+B_3 e^{-(c_1+c_2)t}\right)\sin^2 n\pi x \\ -A_{12}t-A_{13}\sum_{n=1}^{\infty}\left(A_1 t-A_2 e^{-c_2 t}\right)\sin n\pi x\end{array}\right) \\ +a_9 A_9\sum_{n=1}^{\infty}A_2\sin n\pi x-a_8\left(\begin{array}{l} A_{10}\sum_{n=1}^{\infty}\frac{A}{c_1}\sin n\pi x+A_{11}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\left(B_4\right)\sin^2 n\pi x \\ +A_{13}\sum_{n=1}^{\infty}A_2\sin n\pi x\end{array}\right)\end{array}\right) \tag{16}$$

$$\phi(x,t)=\sum_{n=1}^{\infty}Ae^{-c_1t}\sin n\pi x+v\left(\sum_{n=1}^{\infty}\left(\begin{array}{l}-2A_{49}\sum_{n=1}^{\infty}Ate^{-c_1t}-2A_{50}\sum_{n=1}^{\infty}\left[\frac{A_1}{c_1}+A_{53}e^{-c_2t}-A_{54}e^{-c_1t}\right]\\ -A_{56}\left[1-e^{-c_1t}\right]-2A_{51}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}A_{52}\left[\begin{array}{l}AA_1te^{-c_1t}-A_{55}e^{-(c_1+c_2)t}\\ +A_{55}e^{-c_1t}\end{array}\right]\\ -2A_{42}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}A_{52}\left[\begin{array}{l}\frac{A_1^2}{c_1}+A_{57}e^{-c_2t}+A_{58}e^{-2c_2t}-\frac{A_1^2}{c_1}e^{-c_1t}\\ -A_{57}e^{-c_1t}-A_{58}e^{-c_1t}\end{array}\right]\\ -2A_{43}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\left[\begin{array}{l}AA_1^2te^{-c_1t}-A_{59}e^{-(c_1+c_2)t}\\ -A_{59}e^{-(c_1+c_2)t}-A_{60}e^{-(c_1+2c_2)t}\\ +A_{59}e^{-c_1t}+A_{59}e^{-c_1t}+A_{60}e^{-c_1t}\end{array}\right]\\ +2A_{44}\sum_{n=1}^{3}\sum_{n=1}^{3}A_{61}\left[1-e^{-c_1t}\right]-2A_{45}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}A_{52}\left[\begin{array}{l}AA_1te^{-c_1t}\\ -A_{55}e^{-(c_1+c_2)t}\\ +A_{55}e^{-c_1t}\end{array}\right]\\ +2A_{46}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}\left[A_{62}e^{-2c_1t}+A_{63}e^{-(2c_1+c_2)t}-A_{62}e^{-c_1t}-A_{63}e^{-c_1t}\right]\end{array}\right)\sin n\pi x\right) \qquad (17)$$

$$\left.\begin{array}{l}\theta(x,t)=\left(\sigma_1+\sum_{n=1}^{\infty}\left(A_1+(b_n-A_1)e^{-c_2t}\right)\sin n\pi x\right)+\\ v\left(\sum_{n=1}^{\infty}\left(\begin{array}{l}-2\frac{A_{72}}{c_2}\left[1-e^{-c_2t}\right]-2A_{73}\sum_{n=1}^{\infty}\left[\frac{A_1}{c_2}+(b_n-A_1)te^{-c_2t}-\frac{A_1}{c_2}e^{-c_2t}\right]\\ -2A_{68}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}A_{52}\left[\begin{array}{l}\left[1-e^{-c_2t}\right]\frac{A_1^2}{c_2}+2A_1(b_n-A_1)te^{-c_2t}\\ +\left[e^{-c_2t}-1\right]\frac{(b_n-A_1)^2}{c_2}\end{array}\right]\\ +2A_{69}\sum_{n=1}^{\infty}\frac{A}{(c_2-c_1)}\left[e^{-c_1t}-e^{-c_2t}\right]+2A_{70}\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}A_{52}\left[\begin{array}{l}\frac{AA_1}{(c_2-c_1)}e^{-c_1t}-\frac{A(b_n-A_1)}{c_1}e^{-(c_2+c_1)t}\\ -\frac{AA_1}{(c_2-c_1)}e^{-c_2t}+\frac{A(b_n-A_1)}{c_1}e^{-c_2t}\end{array}\right]\end{array}\right)\sin n\pi x\right)\end{array}\right\} \quad (18)$$

where;

$$
\left(\begin{array}{l}
c_1=\left(\beta_1+D_1(n\pi)^2\right),\ A=\dfrac{2\left[1-(-1)^n\right]}{n\pi},\ b_1=\left(4R_a\in+\alpha_1\right),\ b_2=\left(\sigma_1\left(4R_a\in+\alpha_1\right)+\left(R_a+\alpha_1\gamma_1\right)\right),\\
c_2=\left(b_1+\lambda_1(n\pi)^2\right),\ A_1=\dfrac{2b_2\left[(-1)^n-1\right]}{n\pi c_2},\ A_2=\left(\dfrac{b_n-A_1}{c_2}\right),\ A_3=a_6f(e-2),\ A_4=r(e-2),\\
A_5=\dfrac{1}{2}\in,\ A_6=\in\left(r(e-2)\right)\sigma_1,\ B=\dfrac{b_n-A_1}{2},\ B_1=\left(2A_1+B\right),\ A_7=\left(A_4+A_5+A_6\right),\ b_n=\dfrac{2\sigma_1}{n\pi}\left[(-1)^n-1\right]\\
A_8=\left(1+f(e-2)\sigma_1\right),\ A_9=f(e-2),\ A_{10}=\left(1+(e-2)\sigma_1\right),\ A_{11}=(e-2),\ B_2=\dfrac{AA_1}{c_1},\\
B_3=\dfrac{A(b_n-A_1)_1}{c_1+c_2},\ A_{12}=\left(1+(e-2)\sigma_1\right)q,\ A_{13}=(e-2)q,\ B_4=\left(B_2+B_3\right),\ A_{14}=\left(1+f(e-2)\sigma_1\right)q,\\
A_{15}=f(e-2)q,\ A_{16}=\left(1+r(e-2)\sigma_1\right),\ A_{17}=r(e-2),\ A_{18}=\left(1+r(e-2)\sigma_1\right)q, A_{19}=r(e-2)q,\\
A_{20}=\dfrac{1}{2}\in\left(\left(1+r(e-2)\sigma_1\right)\sigma_1\right),\ A_{21}=\dfrac{1}{2}\in r(e-2)\sigma_1,\ A_{22}=\dfrac{1}{2}\in\left(\left(1+r(e-2)\sigma_1\right)\sigma_1q\right),\\
A_{23}=\dfrac{1}{2}\in r(e-2)\sigma_1q,\ A_{24}=\dfrac{1}{2}\in\left(1+r(e-2)\sigma_1\right),\ A_{25}=\dfrac{1}{2}\in r(e-2),\ A_{26}=\dfrac{1}{2}\in\left(1+r(e-2)\sigma_1\right)q,\\
A_{27}=\dfrac{1}{2}\in r(e-2)q,\ A_{28}=(p+q)\left(1+(e-2)\sigma_1\right),\ A_{29}=(p+q)(e-2),\ A_{30}=(pq)\left(1+(e-2)\sigma_1\right),\\
A_{31}=(pq)(e-2), A_{32}=\left(a_1A_{14}+a_2A_{18}+a_2A_{22}+a_3A_{30}\right),\ A_{33}=\left(a_1A_{16}+a_2A_{20}\right),\ A_{34}=\left(a_2A_{19}+a_2A_{23}+a_2A_{26}\right),\\
A_{35}=\left(a_2A_{17}+a_2A_{21}+a_2A_{22}+a_2A_{24}\right),\ A_{36}=\dfrac{1}{2}a_1A_8,\ A_{37}=\dfrac{1}{2}a_1A_{15},\ A_{38}=\dfrac{2}{3}a_1A_9,\ A_{39}=\dfrac{1}{2}A_{33},\ A_{40}=\dfrac{1}{2}A_{34},\\
A_{41}=\dfrac{2}{3}A_{35},\ A_{42}=\dfrac{2}{3}a_2A_{27},\ A_{43}=\dfrac{3}{8}a_2A_{25},\ A_{44}=\dfrac{2}{3}a_3A_{10},\ A_{45}=\dfrac{2}{3}a_3A_{29},\ A_{46}=\dfrac{3}{8}a_3A_{11},\ A_{47}=\dfrac{1}{2}a_3A_{38},\\
A_{48}=\dfrac{1}{2}a_3A_{31},\ A_{49}=\left(A_{36}+A_{39}+A_{47}\right),\ A_{50}=\left(A_{37}+A_{40}+A_{48}\right),\ A_{51}=\left(A_{38}+A_{41}\right),\ A_{52}=\left[\dfrac{1-(-1)^n}{n\pi}\right],\\
A_{53}=\dfrac{b_n-A_1}{c_1-c_2},\ A_{54}=\dfrac{A_1\left(c_1-c_2\right)+c_1\left(b_n-A_1\right)}{c_1\left(c_1-c_2\right)},\ A_{55}=\dfrac{A\left(b_n-A_1\right)}{c_2},\ A_{56}=2\dfrac{A_{32}A_{52}}{c_1},\ A_{57}=\dfrac{2A_1\left(b_n-A_1\right)}{c_1-c_2},\\
A_{58}=\dfrac{\left(b_n-A_1\right)^2}{\left(c_1-2c_2\right)},\ A_{59}=\dfrac{AA_1\left(b_n-A_1\right)}{c_2},\ A_{60}=\dfrac{A\left(b_n-A_1\right)^2}{2c_2},\ A_{61}=\dfrac{A_{52}A^2}{c_1},\ A_{62}=\dfrac{A^2A_1}{c_1},\ A_{63}=\dfrac{A^2\left(b_n-A_1\right)}{\left(c_1+c_2\right)},\\
A_{64}=\left(a_4A_{16}+a_4A_{20}-a_5A_{12}\right),\ A_{65}=\dfrac{a_4A_{17}}{2},\ A_{66}=\dfrac{a_4A_{21}}{2},\ A_{67}=\dfrac{a_4A_{24}}{2},\ A_{68}=\dfrac{2a_4A_{25}}{3},\ A_{69}=\dfrac{a_5A_{10}}{2},\\
A_{70}=\dfrac{2a_5A_{11}}{3},\ A_{71}=\dfrac{a_5A_{13}}{2},\ A_{72}=\left(A_{64}A_{52}\right),\ A_{73}=\left(A_{65}+A_{66}+A_{67}-A_{71}\right)
\end{array}\right)
$$

The computation were done using Maple 17 to generate the graphs.

Results and Discussion

To conclude this analysis we examine the effect of Frank-Kamenetskii number(δ), Radiation number(R_a), Peclet energy number (P_e) and Activation energy number$(\in)$ on temperature $\theta(x,t)$. Analytical solution given by equation (14)—(18), is computed using computer symbolic algebraic package MAPLE 17. The numerical results obtained from the method are shown in Figures 1, 2, 3 and 4.

Figure 1 depicts the graph of temperature $\theta(x,t)$ against distance x for different values of Frank-Kamenetskii number (δ). It is observed that the temperature decreases but later increases along the distance and the minimum temperature decreases as Frank-Kamenetskii number increases.

Figure 2 depicts the graph of temperature $\theta(x,t)$ against distance x for different values of Radiation number (R_a). It is observed that the temperature decreases but later increases along the distance and the minimum temperature decreases as Radiation number increases.

Figure 3 depicts the graph of temperature $\theta(x,t)$ against distance x for different values of Peclet energy number (P_e). It is observed that the temperature decreases but later increases along the distance and the minimum temperature decreases as Peclet energy number increases.

Figure 4 depicts the graph of temperature $\theta(x,t)$ against distance x for different values of dimensionless activation energy number $(\in)$. It is observed that the temperature decreases but later increases along the distance and the minimum temperature decreases as dimensionless activation energy number increases.

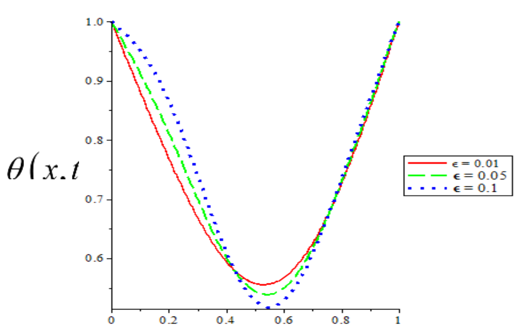

Figure 1: Graph of temperature $\theta(x,t)$ against distance x for different values of Frank-Kamenetskii number (δ).

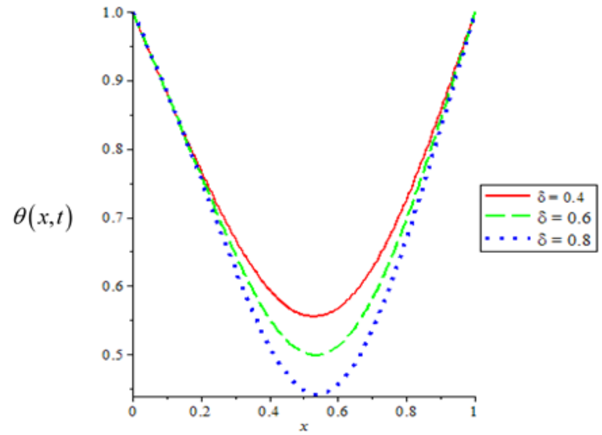

Figure 2: Graph of temperature $\theta(x,t)$ against distance χ for different values of Radiation number (R_a).

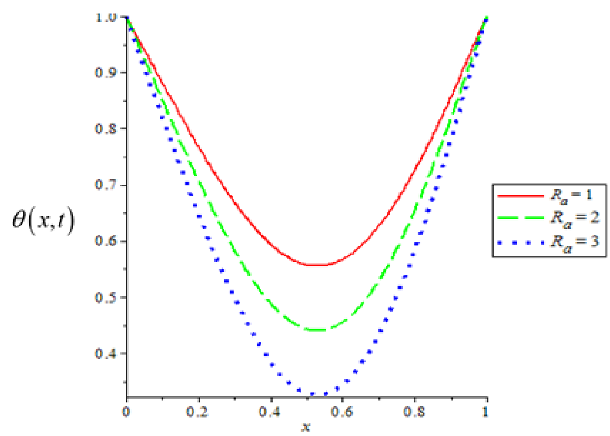

Figure 3: Graph of temperature $\theta(x,t)$ against distance χ for different values of Peclet energy number (P_e)

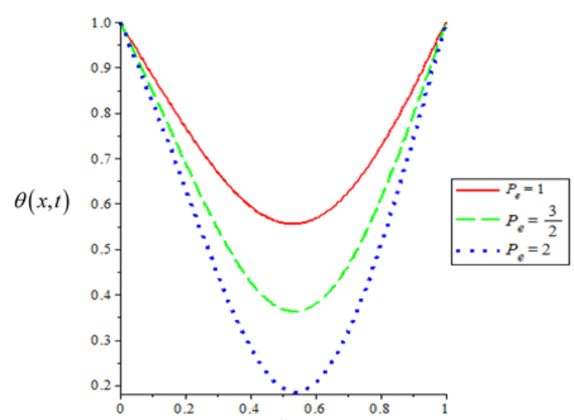

Figure 4: Graph of temperature $\theta(x,t)$ against distance χ for different values of dimensionless activation energy number $(\in)$.

We observed from the discussions that increment in all the four parameters reduced the temperature, as shown or demonstrated in Figures 1, 2, 3 and 4. Therefore, it is important for fire safety precaution. Wildland fire could be suppress taking into account the effects of these parameters as it relates to temperature.

Conclusion

For a high activation energy situation (i.e. as $\in \longrightarrow \mathbf{O}$), we have solved the equations governing the fire spread model using direct integration and eigenfunction expansion technique. From the results obtained, we can conclude that, Frank-Kamenetskii number, Radiation number, Peclet energy number and Activation energy number all decreased the temperature.

This results obtained are not only expected to guide fire fighters to combat or suppress fire outbreak but to also manage the danger associated with forest fire spread.

References

Carlos, B. S. (2014). A comprehensive methodology to predict forest fires behavior using complementary models. PhD thesis, Universitat Autonoma de Barcelona

Goldammer, J. & Ronde, C. (2004). Wildland Fire Management Handbook for Sub- Sahara Africa. *Global Fire Monitoring Center.*

Johnson, E. A., and Miyanishi, K., eds. (2001). Forest fires: Behavior and ecological effects. Academic Press, San Diego, CA, 594 pp.

Pastor, E., Zarate, L., Planas, E. & Arnaldos, J. (2003). Mathematical models and calculation systems for the study of wildland fire behaviour. *Progress in Energy and Combustion Science,* 29(2),139–153.

Perminov, V. A. (2018). Mathematical modelling of wildland fires initiation and spread using a coupled atmospheric-forest fire setting, *The Italian Association of Chemical Engineering Online at www.aidic.it/cet* doi: 10.3303/CET1870292 ISBN978-88-95608-67-9; ISSN 2283-9216.