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Optimal Design of Rectifying Double Sampling Plan with Inspection Errors

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ABSTRACT

In this paper, effect of type I and type II inspection errors on the performance measures of rectifying double sampling (RDS) plan is investigated. An economic model to determine the optimal sampling plan developed by Hsu and Hsu (2012) is extended to rectifying double sampling (RDS) plan and modified to incorporate inspection error. Optimal sampling plan that minimize the total cost and satisfied both the producer's and the consumer's risk requirement in both the existing model and the modified model is determined. Comparison between the optimal sampling plans of RDS plan in the existing model and the modified model showed that the optimal RDS plan in the modified model exhibited higher probability of acceptance of 0.9030, smaller sample sizes of n1=10, n2=20, Average Total Inspection of 113.69 and Total cost of 448.45. Whereas the existing model showed lower probability of acceptance of 0.8891, bigger sample sizes of n1=96, n=192. Average Total Inspection of 237.32 and Total cost of 534.43. It is therefore found that the modified model performed better and is more economical in terms of cost than the existing model.

Keywords: Average Outgoing Quality (AOQ), Average Total Inspection (ATI), producer's risk, Consumer's risk.

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INTRODUCTION

According to Mishra and Sandilya (2009), Acceptance Sampling is a process of evaluating a portion of the product/ material in a lot for the purpose of accepting or rejecting the lot on the basis of conforming or not conforming to quality specifications. According to Amitava (2016), Acceptance sampling can be performed during inspection of incoming raw materials, components, and assemblies, in various phases of in-process operations, or during final product or service inspection. It can be used as a form of product inspection between companies and their vendors, between manufacturers and their customers. Acceptance sampling plan is classified as either by variable or attributes.

In acceptance sampling by variable quality characteristics is measured using numerical value while Attributes are quality characteristics that are expressed on a "go, no-go" basis (Montgomery 2009). Rectifying inspection is another form of acceptance sampling carried out on the rejected lot during sampling inspection. The rejected lots are subjected to one-hundred percent and all the detected defective units are repaired or replaced with non-defective units. Sampling inspection is never 100% reliable and involves two types of errors known as type I and Type II inspection errors. Type I inspection error occurs when non-defective unit is misclassified as defective, while type II inspection error occurs when a defective units is misclassified as non-defective unit. The probability of rejecting a lot with acceptable quality level (AQL) due to type I error is known as producer's risk. The probability of accepting a lot with quality level equal or higher than LTPD is known as consumer's risk

Jalbout *et al* (2002) investigated the impact of inspection error on the Average Outgoing Quality (AOQ) and Operating characteristics (OC) curve in an industrial process Yinfeng and Loon (2006) investigated the effect of constant inspection errors on general chain sampling scheme using the performance measures such as operating characteristic function, average total inspection and average outgoing quality Nezhad and Nasab (2011) developed a cost model for acceptance sampling problem with objective of finding a constant control level that minimizes the total cost of rejection, cost of inspection and the cost of accepting defective units. Subramaniam and Yuvara (2011) proposed a model to minimize average cost over different combinations of Bayesian acceptance sampling plans with regards to the cost of accepting defective units in order to achieve optimal Bayesian Single Sampling Plan for attribute. Valtteri (2012) developed a cost model for comparing different inspection strategies and for creating an understanding of the structure of the costs of bad quality in automotive manufacturing. The model points out the fact that choosing a correct inspection strategy for quality control will lead to a significant increase in the profit margin of the business. Castillo-Villar *et al.* (2012) developed a model for supply-chain design that considers the Cost of Quality as well as the traditional manufacturing and distribution costs (SC-COQ model. Anuja *et al.* (2013) provided a cost effective solution for appropriate sampling plan to minimize cost of inspection and maintain quality of product using novel ABCDE classification of product so that different categories of products follow optimum plan of sampling. Fallahnezhad *et al.* (2014) presented an optimal iterative decision rule for minimizing total cost in designing a sampling plan for machine replacement problem using the approach of dynamic programming. Mohammed *et al.* (2015) developed a mathematical model to design a single stage and double stage sampling plans used to determine the optimal tolerance limits and sample size. Ching-ho *et al*., (2015) developed an economic cost model for variable acceptance sampling that minimizes quality cost in lots manufacturing. Muhammad and Chang (2016) developed a model to reduce inspection cost in acceptance single sampling plan by determining the optimal number of quality inspectors with respect to their skill levels using goal programming. Hagenimana *et al.* (2016) developed a model for determining the appropriate level of inspection sampling for manufacturers which considers the interest of the consumers who wish to minimize cost of production while ensuring the final product is of high quality. Chen *et al*. (2016) developed a model with optimal sampling strategy that can be used in acceptance single sampling to reduce the introduction of damaging pests on agricultural imports. Nirmala *et al*. (2016) developed a model for finding optimum single sampling plans based on prior binomial distribution by minimizing the average acceptance cost such that the cost of accepting a defective unit is less and both producer's and consumer's risks are minimized. Fallahnezhad and Ahmadi (2016) presented an optimization model for designing an acceptance sampling plan based on cumulative sum of run length of conforming units. The objective is to minimize the total losses for both the producer and the consumer. Fallahnezhad and Qazvini (2016) presented a new economical scheme of the acceptance sampling plan in a two-stage approach based on the Maxima Nomination Sampling technique. Muhammad *et al*. (2017) developed a cost model for the evaluation of inspection strategies in manufacturing system to minimize cost and maximize quality of products. Namin .

(2017) extended the model of Hsu and Hsu (2012) to a multi-objective economic-statistical design (MOESD) for acceptance single sampling plan to strike a balance between cost and quality features.
In this paper, Hsu and Hsu (2012) economic single-sampling plan model is extended to rectifying double sampling plan and modified to incorporate inspection error. The modified model maximizes the probability of lot acceptance with acceptable quality level (AQL) and minimizes the probability of lot acceptance with lot tolerant percent defective (LTPD). Optimal sampling plan to optimize the total cost is determined and comparison of the performances of their model and the modified model is made.

2.0 MATERIALS AND METHODS

2.1 Apparent Fraction Defective (p_e)

Apparent (Observed) Fraction defective p_e is the fraction of incoming lot which is observed as defective by the inspector. Let p be the true fraction defective and is written as:

$$p_e = p(1-e_2) + (1-p)e_1 \quad (1)$$

e_1: Is type I inspection error (where a non-defective unit is classified as defective)
e_2: Is type II inspection error (where a defective unit is classified as non-defective unit).

2.2 Acceptance Double Sampling plan

A random sample of size n is randomly taken from a lot size of N, the number of defective units in the first sample x_1 is compared with the first acceptance number c_1 If $x_1 \leq c_1$ the lot is accepted but if $x_1 > c_2$ the lot is rejected. However, if x_1 lies between c_1 and c_2 a second sample n_2 is taken. If $x_1 + x_2 \leq c_1$ the lot is accepted but if $x_1 + x_2 > c_2$ the lot is rejecte

Probability of acceptance P_a in double sampling plan under error-free inspection assumption is obtained by adding probability of acceptance on the first sample and on the second sample as given below:

$$P_a = P_{a_1} + P_{a_2} = \sum_{x_1=0}^{c_1} \binom{n_1}{x_1} p^{x_1}(1-p)^{n_1-x_1} + \sum_{x_1=c_1+1}^{c_2} \left\{ \left[\binom{n_1}{x_1} p^{x_1}(1-p)^{n_1-x_1} \right] \times \left[\sum_{x_2=0}^{c_2-x_1} \binom{n_2}{x_2} p^{x_2}(1-p)^{n_2-x_2} \right] \right\}$$
(2)

The probability of acceptance of the lot when inspector error is considered (P_{a_e}) is calculated as:

$$(P_{a_e}) = \sum_{x_1=0}^{c_1} \binom{n_1}{x_1} p_e^{x_1}(1-p_e)^{n_1-x_1} + \sum_{x_1=c_1+1}^{c_2} \left\{ \left[\binom{n_1}{x_1} p_e^{x_1}(1-p_e)^{n_1-x_1} \right] \times \left[\sum_{x_2=0}^{c_2-x_1} \binom{n_2}{x_2} p_e^{x_2}(1-p_e)^{n_2-x_2} \right] \right\}$$
(3

2.3 Rectifying Double Sampling (RDS) Plan

When rectifying inspection is carried out on acceptance double sampling plan, all the defective units in the sample and in the rejected lot is subjected to 100% inspection where all the observed defective units are replaced with non-defective units.

Average Outgoing Quality (AOQ) represents the average quality of stream of lots after rectifying inspection. Average outgoing quality limit (AOQL) is the maximum average quality of stream of lots that leave the inspection station after rectifying inspection.

The AOQ for Double Sampling Plan when inspection error is considered is given as:

$$AOQ_{1e} = \frac{p(n_1e_2) + p(N-n_1)P_{a1_e} + p(N-n_1)(1-P_{a_{1e}})e_2 + p(n_2e_2) + p(N-n_1-n_2)P_{a2_e} + p(N-n_1-n_2)e_2(1-P_{a_{2e}})}{N}$$
(4)

The average total inspection (ATI) is the average number of units inspected per lot.
The ATI for double sampling plan under inspection with no error assumption is calculated as:

$$\text{ATI} = n_1P_{a_1} + (n_1+n_2)P_{a2} + N(1-P_{a1}-P_{a2}) \quad (5)$$

The Average Total Inspection for double sample inspection when inspector error is considered is:

$$ATI_e = n_1 P_{a_{1e}} + (n_1 + n_2)P_{a2e} + N(1 - P_{a_{1e}} - P_{a_{2e}}) \quad (6)$$

2.4 Proportion of Defective Units in Rectifying Double Sampling (RDS) Plan

In rectifying double sampling plan, some defective units are accepted in uninspected portion of the accepted lot while some defective units are detected and removed during 100% inspection of rejected lot. Defective units in accepted lots denoted as (Dn) is given as:

$$Dn = p\left[P_{a_1}(N - n_1) + P_{a_2}(N - n_1 - n_2)\right] \quad (7)$$

The defective units detected denoted as (Dd) come from the defective units discovered either in sampling or 100% inspection of the rejected lots. Montgomery (2009)

$$Dd = pn_1 + p(N - n_1)(1 - Pa_1) + pn_2 + p(N - n_1 - n_2)(1 - P_{a2}) \quad (8)$$

When inspection error is considered, some defective units are classified as not being defective but are actually defective while some defective units that are actually defective are classified as being defective.

Therefore proportion of defective units misclassified as not being defective is denoted as (Dn_e) and is given below:

$$Dn_e = pn_1e_2 + p(N - n_1)P_{a1_e} + p(N - n_1 - n_2)P_{a2_e} + pn_2e_2 + p(N - `n_1)(1 - P_{a_{1e}})e_2 + p(N - n_1 - n_2)e_2(1 - P_{a_{2e}}) \quad (9)$$

The proportion of defective units that are actually defective and are classified as being defective during 100% inspection of the rejected lot is denoted as (Dd_e) and is given as:

$$Dd_e = pn_1(1 - e_2) + p(N - n_1)(1 - P_{1ae})(1 - e_2) + pn_2(1 - e_2) + p(N - n_1 - n_2)(1 - P_{2ae})(1 - e_2) \quad (10)$$

2.5 Probability of detecting defective unit in a sample

If one or more defective units are detected in the sample, the probability of observing one or more defective units in the sample is:

$$P(x \geq 1) = 1 - P(x = 0) = 1 - (1 - p)^n \quad (11)$$

Lot proportion defective according to Fallahnezhad *et al* (2018) is then given as:

$$p = 1 - (1 - p)^n \quad (12)$$

2.6 Apparent Proportion Defective (p_e)

Let

$e_1 = P\{unit\ is\ classified\ as\ defective | the\ unit\ is\ non-defective\}$

$e_2 = \text{P}\{\text{unit is classified as non} - \text{defective}|\text{the unit is defective}\}$

We use the following to express the apparent proportion defective p_e:

A: The event in which a unit is defective

B: The event which a unit is classified as defective

Apparent fraction defective p_e is thus obtained as:

$$p_e = P(B) = P(B|A)P(A) + P(B|A')P(A')$$

$$p_e = (1 - e_2)p + (1 - p)e_1 \quad (13)$$

$$p_e = 1 - (1 - p)^n(1 - e_2) + e_1(1 - p)^n \quad (14)$$

2.7 Producer's risk and Consumer's risk in the Design of Rectifying Double Sampling plan given AQL and LTPD quality levels

Consider the producer's risk (α) and its associated quality level$p_1 = AQL$ as well as the consumer's risk (β) with its associated quality level $p_2 = LTPD$,we formulate the probabilities of lot acceptance $1 - \alpha$ with fraction defective $p_1 = AQL$ and β with fraction defective $p_2 = LTPD$ for RDS plan as follows:

The Probability of lot acceptance$(1 - \alpha)$ with proportion defective $p_1 = AQL$ under error-free inspect ion is given as:

$$1 - \alpha = P_a = P_{a_1} + P_{a_2} = P\left(x_1 \leq c_1 | n_1, p_1 = AQL\right) + P\left(x_1 + x_2 \leq c_2 | c_1 < x_1 \leq c_2, n_1, n_2, p_1 = AQL\right) \quad (15)$$

$$= \sum_{x_1=0}^{c_1} \binom{n_1}{x_1} AQL^{x_1}(1-AQL)^{n_1-x_1} + \sum_{x_1=c_1+1}^{c_2} \left\{ \left[\binom{n_1}{x_1} AQL^{x_1}(1-\mathrm{AQL})^{n_1-x_1} \right] \times \left[\sum_{x_2=0}^{c_2-x_1} \binom{n_2}{x_2} AQL^{x_2}(1-AQL)^{n_2-x_2} \right] \right\} \quad (16)$$

$$1-\alpha = P_a(AQL) \quad (17)$$

Thus the probability of rejection of lot with acceptable quality level (AQL) or producer's risk is:

$$1-P_a(AQL) = \alpha \quad (18)$$

When inspection error is considered equations (16) becomes:

$$= \sum_{x_1=0}^{c_1} \binom{n_1}{x_1} \{[1-(1-AQL)^{n_1}](1-e_2) + e_1(1-AQL)^{n_1}\}^{x_1}\{(1-\mathrm{AQL})^{n_1}(1-e_2)+e_1(1-AQL)^{n_1}\}^{n_1-x_1} + \sum_{x_1=c_1+1}^{c_2} \left\{ \left[\binom{n_1}{x_1} \{[1-(1-AQL)^{n_1}](1-e_2)+e_1(1-AQL)^{n_1}\}^{x_1}\{(1-\mathrm{AQL})^{n1}(1-e_2)+e_1(1-AQL)^{n1}\}^{n_1-x_1} \right] \times \left[\sum_{x_2=0}^{c_2-x_1} \binom{n_2}{x_2} \{[1-(1-AQL)^{n1}](1-e_2)+e_1(1-AQL)^{n1}\}^{x_2} \{(1-\mathrm{AQL})^{n1}(1-e_2)+e_1(1-AQL)^{n1}\}^{n_2-x_2} \right] \right\} \quad (19)$$

Thus the probability of lot acceptance is:

$$1-\alpha = P_{a_e}(AQL_e) \quad (20)$$

The probability of rejection or producer's risk(α) lot is:

$$1-Pa_e(AQL_e) = \alpha \quad (21)$$

Considering equation (11) the probability of accepting the lot with quality level $p_2 = LTPD$ which is also referred to as consumer's risk (β) under error-free inspection assumption is calculated below:

$$\beta = \sum_{x_1=0}^{c_1} \binom{n_1}{x_1} LTPD^{x_1}(1-LTPD)^{n_1-x_1} + \sum_{x_1=c_1+1}^{c_2} \left\{ \left[\binom{n_1}{x_1} LTPD^{x_1}(1-LTPD)^{n_1-x_1} \right] \times \left[\sum_{x_2=0}^{c_2-x_1} \binom{n_2}{x_2} LTPD^{x_2}(1-LTPD)^{n_2-x_2} \right] \right\} \quad (22)$$

$$\beta = P_a(LTPD \quad (23)$$

When inspection error is considered the probability of acceptance given in eqn (22) becomes:

$$= \sum_{x_1=0}^{c_1} \binom{n_1}{x_1} \{[1-(1-LTPD)^{n_1}](1-e_2)+e_1(1-LTPD)^{n_1}\}^{x_1}\{(1-\mathrm{LTPD})^{n_1}(1-e_2)+e_1(1-LTPD)^{n_1}\}^{n_1-x_1} + \sum_{x_1=c_1+1}^{c_2} \left\{ \left[\binom{n_1}{x_1} \{[1-(1-LTPD)^{n_1}](1-e_2)+e_1(1-LTPD)^{n_1}\}^{x_1}\{(1-LTPD)^{n1}(1-e_2) + e_1(1-LTPD)^{n1}\}^{n_1-x_1} \right] \times \left[\sum_{x_2=0}^{c_2-x_1} \binom{n_2}{x_2} \{[1-(1-LTPD)^{n1}](1-e_2)+e_1(1-LTPD)^{n1}\}^{x_2} \{(1-LTPD)^{n1}(1-e_2) + e_1(1-LTPD)^{n1}\}^{n_2-x_2} \right] \right\} \quad (24)$$

Thus $\beta = P_{ae}(LTPD_e)$ (25)

2.8 Economic Cost Model,

Hsu and Hsu (2012) single sampling cost model as presented below:

Minimize Total cost $(TC) = C_i ATI + C_f Dd + C_o Dn$ (26)

Subject to $1-P_a(AQL) \le \alpha$

$P_a(LTPD) \le \beta$

Where TC is the total cost, C_i is the cost of inspection per unit,C_f is the internal failure cost (which include cost of repair, scrap or rework of defective unit) and C_o is the external failure cost or post sales cost (which include repair or replacement cost). α and β represent producer's and consumer's risk respectively.

2.9 Modified Economic Cost Model

Hsu and Hsu (2012) acceptance single sampling model is extended to double sampling plan and modified to incorporate inspection error. Multi-objective functions are also introduced. The modified model is as presented below:

Minimize Total Cost $(TC) = C_i ATI_e + C_f Dd_e + C_o Dn_e$ (27)

Maximize $P_{a_e}(AQL_e)$
Minimize $P_{a_e}(LTPD_e)$
Subject to $1 - P_{a_e}(AQL_e) \leq \alpha$
$P_{a_e}(LTPD_e) \leq \beta$

AQL_e and $LTPD_e$ represent apparent (observed) Acceptable Quality Level and apparent (observed) Lot Tolerant Percent Defective respectively. Other parameters with inspection error are as stated above.

3.0 RESULTS AND DISCUSSION

R- Programming Software and MS Word –Excel is used for the analysis and the results are presented in tables and figures below:

Effect of Type I and Type II inspection errors on the AOQ, AOQL and ATI of Rectifying Double Sampling (RDS) Plan

Table 1: Effect of type I inspection error on AOQ of RDS

p	AOQ			
	$(\mathbf{0.00, 0.00})$	$(\mathbf{0.02, 0.00})$	$(\mathbf{0.04, 0.00})$	$(\mathbf{0.06, 0.00})$
0.01	0.0093	0.0071	0.0030	0.0009
0.02	0.0173	**0.0108**	**0.0037**	**0.0011**
0.03	**0.0211**	0.0092	0.0029	0.0009
0.04	0.0194	0.0070	0.0022	0.0007
0.05	0.0148	0.0049	0.0016	0.0005
0.06	0.0101	0.0033	0.0011	0.0004
0.07	0.0065	0.0022	0.0007	0.0003
0.08	0.0041	0.0014	0.0005	0.0002
0.09	0.0026	0.0009	0.0003	0.0001
0.10	0.0016	0.0006	0.0002	0.0000
0.11	0.0009	0.0004	0.0001	0.0000
0.12	0.0006	0.0002	0.0000	0.0000

Note: *Sampling parameters: N=1000,* n_1*=96,* C_1*=3,* n_2*=192,* C_2*=11*

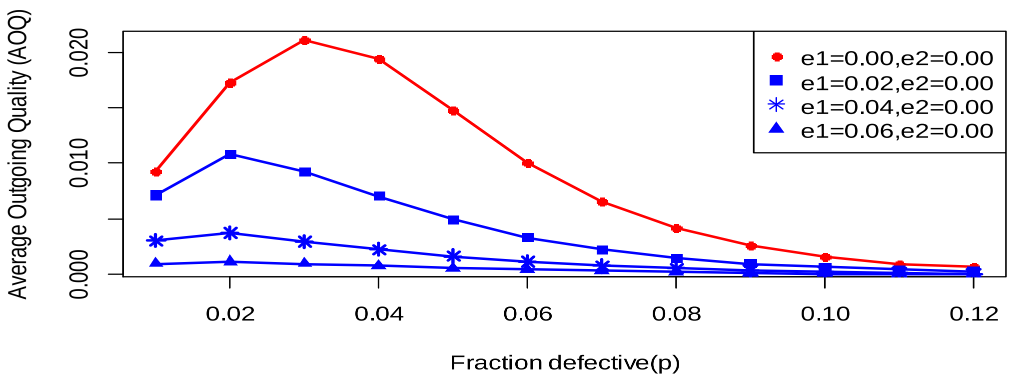

Fig.1:Effect of type I inspection error on AOQ Curve of RDS Plan

From Fig. 1 the AOQ and AOQL for situation when inspection with no error is assumed is uniformly higher than those with some form of type I inspection error. The AOQ and the AOQL however decreased as the type I inspection error increased and type II inspection error is kept at zero. The Average Outgoing Quality Limit (AOQL) is about 0.0211 at p=0.03 when error-free inspection is assumed but decreased to 0.0011 at p =0.02as type I inspection error increased .The decrease in AOQ and AOQL is because type I inspection error increased the probability of rejection of the lot, thus 100% inspection is carried out on the rejected lot where all defective units are replaced with non-defective units.

Table 2: Effect of type II inspection error on AOQ in RSS

	AOQ			
p	$(0.00, 0.00)$	$(0.00, 0.02)$	$(0.00, 0.04)$	$(0.00, 0.06)$
0.01	0.0087	0.0087	0.0087	0.0088
0.02	0.0165	0.0167	0.0168	0.0169
0.03	**0.0208**	**0.0213**	0.0218	0.0222
0.04	0.0199	0.0210	**0.0220**	**0.0230**
0.05	0.0155	0.0170	0.0185	0.0200
0.06	0.0102	0.0120	0.0138	0.0157
0.07	0.0059	0.0078	0.0098	0.0118
0.08	0.0031	0.0051	0.0070	0.0091
0.09	0.0015	0.0035	0.0055	0.0076
0.10	0.0007	0.0028	0.0049	0.0071
0.11	0.0003	0.0025	0.0048	0.0071
0.12	0.0001	0.0025	0.0050	0.0074

Note: *Sampling parameters:* $N=1000$, $n_1=96$, $C_1=3$, $n_2=192$, $C_2=11$

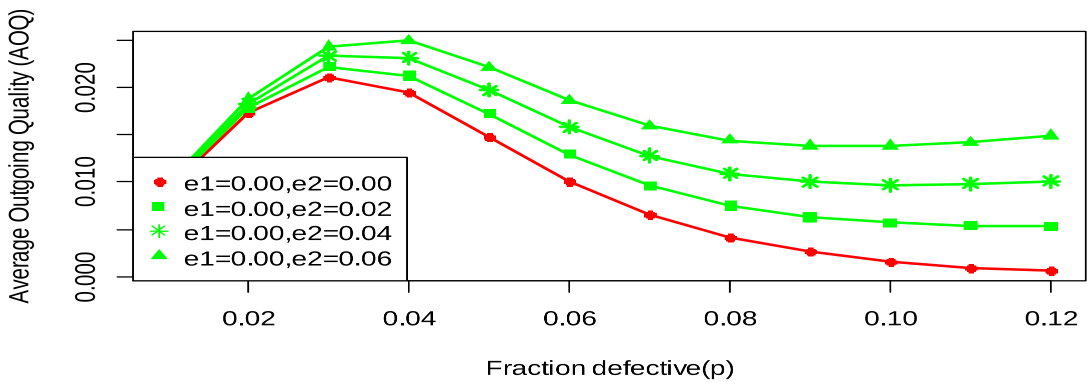

Fig.2:Effect of type II inspection error on AOQ Curve of RDS Plan

From Fig. 2 the AOQ and AOQL for situation when inspection with no error is assumed is uniformly lower than those with some form of type II inspection error. The AOQ and the AOQL however increased as the type II inspection error increased and type I inspection error is kept at zero.

Table 3: Effect of type I inspection error on ATI in RDS

p	ATI			
	$(0.00, 0.00)$	$(0.02, 0.00)$	$(0.04, 0.00)$	$(0.06, 0.00)$
0.01	72.55	293.35	697.98	904.12
0.02	135.32	506.28	824.38	945.14
0.03	297.44	694.73	900.63	968.32
0.04	514.87	824.38	943.83	981.55
0.05	704.42	901.81	967.95	989.23
0.06	832.43	945.14	981.55	993.73
0.07	907.50	969.04	989.36	996.37
0.08	948.88	982.38	993.87	997.92
0.09	971.45	989.95	996.49	998.81
0.10	983.92	994.28	998.88	999.33
0.11	990.93	996.77	998.89	999.63
0.12	994.91	998.19	999.38	999.79

***Note**: Sampling parameters:* $N = 1000, n_1 = 96, C_1 = 3, n_2 = 192, C_2 = 11$

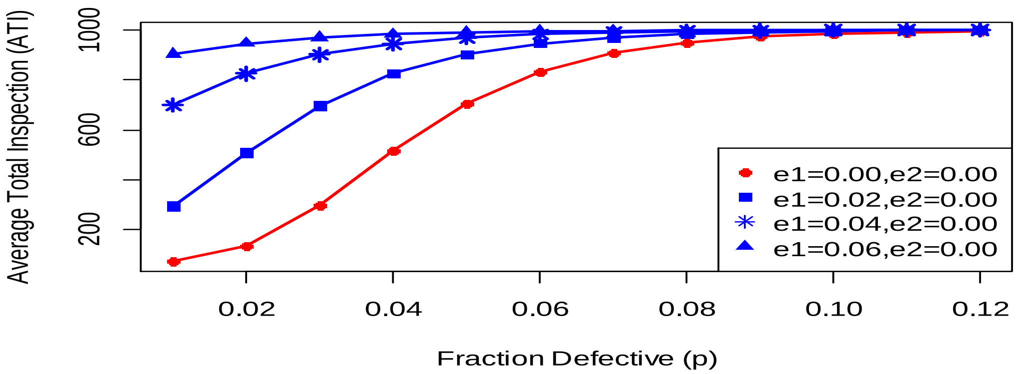

Fig.3:Effect of type I inspection error (e1) on the ATI Curve of RDS Plan

From Fig.3 above, the ATI when error-free inspection is assumed is uniformly lower than those of when the type I inspection error is increased and type II inspection error is kept at zero. The increase in ATI is as a result of 100% inspection carried out on rejected lot due type I inspection error.

Table 4: Effect of type II inspection error on ATI in RDS

p	ATI			
	$(\mathbf{0.00, 0.00})$	$(\mathbf{0.00, 0.02})$	$(\mathbf{0.00, 0.04})$	$(\mathbf{0.00, 0.06})$
0.01	72.55	71.98	71.43	70.89
0.02	135.32	131.02	126.90	122.96
0.03	297.44	285.25	273.28	261.57
0.04	514.34	497.63	480.21	462.65
0.05	704.42	688.13	671.19	653.62
0.06	832.43	820.23	807.23	793.39
0.07	907.50	899.44	890.67	881.16
0.08	948.87	943.83	938.26	932.12
0.09	971.45	968.32	964.83	960.95
0.10	983.92	981.97	979.78	977.32
0.11	990.93	989.72	988.32	986.77
0.12	994.91	994.15	993.28	992.28

***Note**: Sampling parameters:* $N = 1000, n_1 = 96, C_1 = 3, n_2 = 192, C_2 = 11$

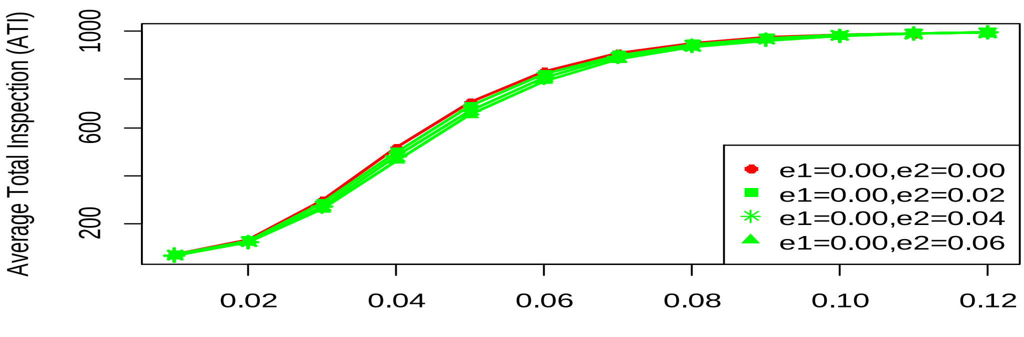

Fig.4:Effect of type II inspection error (e2) on the ATI Curve of RDS Plan

From Fig.4 the ATI when error-free inspection is assumed is uniformly higher than those of when type II inspection error increased and type I inspection error is kept at zero. This is because many defective units are classified as non-defective leading to high probability of acceptance of the lot. There is therefore no 100% inspection carried out on rejected lot hence the reduction in the ATI

3.1 Determination of optimal Sampling Plans for RDS plan in the existing model and the modifi

Table 5: Rectifying Double Sampling (RDS) Plans with error-free Inspection assumption no inspection error satisfying the parameters $AQL = 0.02, LTPD = 0.07, p = 0.03$, α=0.05, β=0.1, with $C_{i,} = 1$,$C_f = 2, C_o = 10$,with n_1 and $n_2 \leq 250$

n_1	n_2	c_1	n_2	AOQ	ATI	D_n	D_d	$1 - P_a(AQL)$	$P_a(LTPD)$	$P_a(p)$	TC
66	132	1	8	0.0226	247.16	22.58	35.43	0.0168	0.0941	0.8717	543.88
67	134	1	8	0.0223	256.78	22.30	35.69	0.0183	0.0867	0.8632	551.13
68	136	1	8	0.022	266.57	22.00	35.96	0.0199	0.0799	0.8546	558.51
69	138	1	8	0.0217	276.53	21.70	36.23	0.0216	0.0736	0.8456	566.03
70	140	1	8	0.0214	286.66	21.40	36.50	0.0235	0.0678	0.8365	573.66
71	142	1	8	0.0211	296.94	21.09	37.78	0.0254	0.0625	0.8270	581.41
72	144	1	8	0.0208	307.35	20.78	37.06	0.0274	0.0576	0.8173	589.27
73	146	1	8	0.0205	317.90	20.46	37.35	0.0295	0.0530	0.8075	597.23
74	148	1	8	0.0201	328.57	20.14	37.64	0.0318	0.0489	0.7973	605.27
75	150	1	8	0.0198	339.35	19.82	37.93	0.0342	0.0450	0.7870	613.41
76	152	1	8	0.0195	350.23	19.49	38.23	0.0367	0.0415	0.7465	621.61
77	154	1	8	0.0192	361.19	19.16	38.53	0.0393	0.0383	0.7658	629.88
77	154	2	8	0.0215	282.40	21.53	36.16	0.0300	0.0953	0.8146	570.02
78	156	1	8	0.0188	372.22	18.83	38.83	0.0420	0.0353	0.7550	638.21
78	156	2	8	0.0212	291.74	21.25	36.41	0.0321	0.0901	0.8057	577.06
79	158	1	8	0.0185	383.32	18.50	39.13	0.0448	0.0326	0.7439	646.59
79	158	2	8	0.0210	301.17	20.96	36.67	0.0343	0.0852	0.7966	584.11
80	160	1	8	0.0182	394.48	18.17	39.43	0.0478	0.0301	0.7328	655.00
80	160	2	8	0.0207	310.68	20.68	36.92	0.0366	0.0805	0.7874	591.32
81	162	2	8	0.0204	320.27	20.39	37.18	0.0390	0.0761	0.7780	598.54
82	164	2	8	0.0201	329.92	20.10	37.44	0.0415	0.0719	0.7686	605.82
83	166	2	8	0.0198	339.63	19.81	37.70	0.0441	0.0680	0.759	613.14
84	168	2	8	0.0195	349.39	19.52	37.96	0.0469	0.0643	0.7493	620.49
85	170	2	8	0.0192	359.18	19.22	38.23	0.0497	0.0608	0.7395	627.88
95	190	3	10	0.0221	263.69	22.09	35.06	0.0175	0.0972	0.8488	554.70
96	192	3	10	0.0219	271.07	21.87	35.25	0.0187	0.0926	0.8419	560.26
96	**192**	**3**	**11**	**0.0229**	**237.32**	**22.88**	**24.24**	**0.0093**	**0.0971**	**0.8891**	**534.60**
97	194	3	10	0.0216	278.10	21.64	35.45	0.0200	0.0882	0.8349	565.88
97	194	3	11	0.0227	244.10	22.68	34.41	0.0100	0.0923	0.8835	539.69
98	196	3	10	0.0214	286.10	21.42	35.64	0.0213	0.0841	0.8277	571.56
98	196	3	11	0.0225	250.98	22.47	34.59	0.0108	0.0878	0.8775	544.87

Table 6: Rectifying Double Sampling (RDS) Plans with inspection error satisfying the parameters $AQL = 0.02, LTPD = 0.07, p = 0.03$. α=0.05, β=0.1 with$n_1$ and $n_2 \leq 250$

n_1	n_2	c_1	c_2	AOQ_e	ATI_e	D_{ne}	D_{de}	$1 - P_{ae}(AQL_e)$	$P_{ae}(LTPD_e)$	$P_{ae}(p)$	TC
9	18	1	8	0.0257	153.68	25.73	34.00	0.0164	0.0714	0.8633	479.01
10	20	2	8	0.0224	266.31	22.39	37.31	0.0437	0.0539	0.7457	564.81
10	**20**	**2**	**10**	**0.0269**	**113.69**	**26.92**	**32.78**	**0.0076**	**0.0800**	**0.9030**	**448.45**
11	22	2	10	0.0225	262,44	22.50	17.17	0.0302	0.0211	0.7541	561.79
11	22	3	10	0.0245	196.46	24.46	35.21	0.0214	0.0717	0.8163	511.49
11	22	3	11	0.0262	137.86	26.20	33.47	0.0098	0.0763	0.8769	466.82
12	24	3	11	0.0216	291.88	21.63	38.01	0.0353	0.0258	0.7220	584.18
13	26	4	13	0.0231	241.35	23.13	36.48	0.0210	0.0320	0.7731	545.59
14	28	4	14	0.0198	353.55	19.80	39.78	0.0365	0.0099	0.6613	631.07

Tables 5 and 6 represent the sampling plans satisfying the stated conditions in the two models stated above. The existing model show optimal sampling plan of$n_1 = 96, n_2 = 192, C_1 = 3, n_2 = 11$ with minimum total cost of 534.60. In the modified model the optimal sampling plan is $n_1 = 10, n_2 = 20, C_1 = 2, C_2 = 10$ with minimum total cost of 448.45. It is noticed that the sample size, Average Total Inspection (ATI), the producer's risk$(1 - P_{ae}(AQL_e))$, the consumer's risk$(P_{ae}(LTPD_e))$ and the total cost (TC) in the optimal sampling plan of the modified model are smaller than in the existing model. This means that the modified model is better and more economical than the existing model.

Table 7: Comparison of the probability of acceptance of optimal sampling plans of RDS plans under the existing model and the modified model

$\alpha = 0.05$		β= 0.1	
Existing model(optimal RDS plan): $n_1 = 96, c_1$=3,$n_2 = 192, c_2 = 11$		modified model (optimal RDS plan): $n_1 = 10, c_1$=2,$n_2 = 20, c_2 = 10$	
p	P_a(p)	p	Pa_e(p)
0.01	1.0000	0.01	1.0000
0.02	**0.9907**=P_a(AQL)	**0.02**	**0.9924**= Pa_e(AQLe)
0.03	0.8893	0.03	0.9030
0.04	0.6420	0.04	0.6647
0.05	0.3763	0.05	0.3851
0.06	0.1955	0.06	0.1852
0.07	**0.0971**=P_a(LTPD)	**0.07**	**0.0800**= Pa_e(LTPDe)
0.08	0.0475	0.08	0.0334
0.09	0.0229	0.09	0.0142
0.1	0.0107	0.1	0.0062

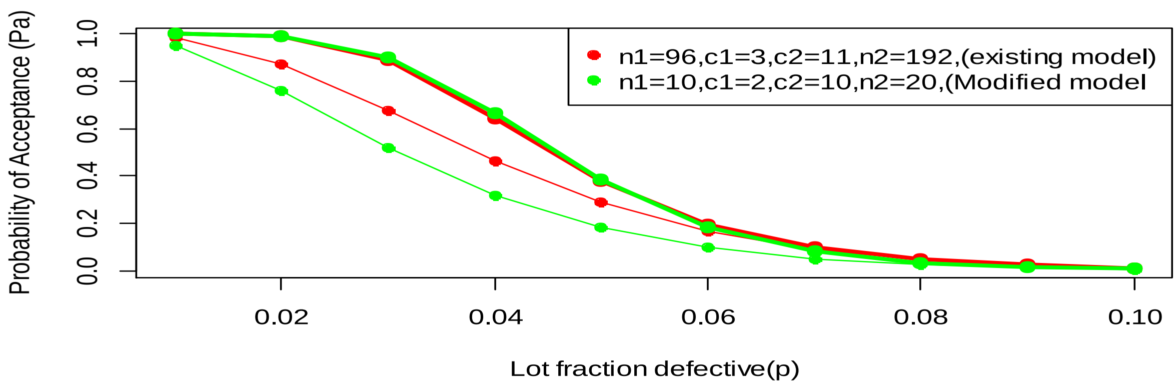

Fig.5:Oc curves for optimal RDS plans in existing model and the Modified Model

The thin curves in the operating characteristics (OC) curve in fig. 5 above represent probability of acceptance on the first sample while the thick curves represent the combined probability of acceptance on the first and second samples. The probability of acceptance for RDS plans in the two models decreased as the fraction defective units of the lot increased. However, the combined probability of acceptance for optimal RDS plan in the modified model is higher at $AQL \leq 0.02$ and lower at $LTPD \geq 0.07$ compared to the combined probability of acceptance in the existing model. The probability of rejecting good lot with acceptable quality level $1 - P_a(AQL)$ or producer's risk (α) in the existing model is 0.93% while the probability of accepting bad lot with unacceptable quality level or consumer's risk (β)= $P_a(LTPD)$ is 9.71%. In the Modified model, the probability of rejecting good lot with acceptable quality level $1 - P_{ae}(AQL_e)$ or producer's risk (α) is 0.76% and the probability of accepting bad lot with unacceptable quality level $P_{ae}(LTPD_e)$ or consumer's risk (β) is 8%.

Since the producer's risk and the consumer's risk in the modified model is lower than in the existing model, it means that the modified model performed better and provides more protection to both the producer and the consumer than the existing model

Table 8: Comparison of the Average Total Inspection (ATI) of optimal RDS plans under the existing model and the modified model

optimal RDS plan(existing model) $n_1 = 96, c_1=3, n_2 = 192, c_2 = 11$		optimal RDS plan(modified model) $n_1 = 10, c_1=2, n_2 = 20, c_2 = 10$	
p	ATI	p	ATI
0.01	99.10	0.01	10.97
0.02	126.99	**0.02**	22.19
0.03	237.32	0.03	113.69
0.04	454.21	0.04	348.83
0.05	676.94	0.05	622.83
0.06	829.04	0.06	818.36
0.07	913.65	**0.07**	921.36
0.08	957.32	0.08	967.07
0.09	979.38	0.09	986.02
0.1	990.31	0.1	993.91

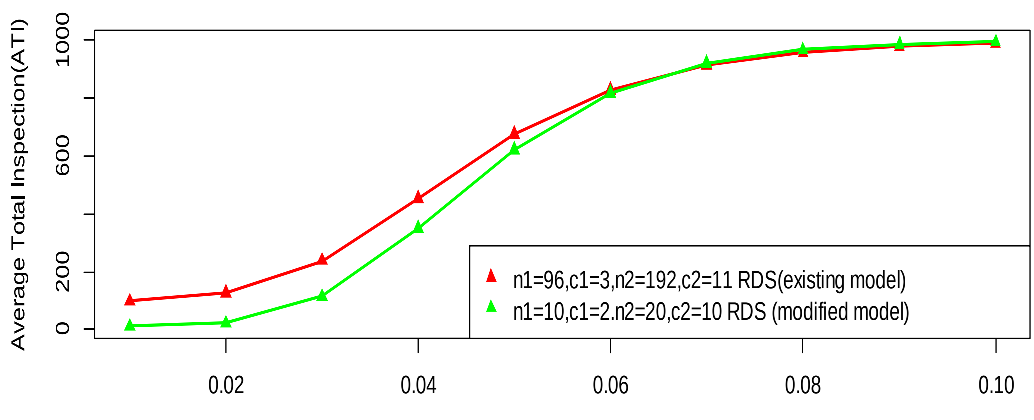

Fig.6:Average Total Inspection (ATI) for optimal RDS under the existing model and the modified model

Tables 5 and 6 above show high ATI of 237.32 for optimal RDS plan in the existing model and low ATI of 113.69 in the modified model. The Average Total Inspection (ATI) for all the sampling plans generally increased as the fraction defective (p) increased as shown in table 8 and fig. 6 above .However, Average Total Inspection (ATI) in the modified model is lower than the ATI in the existing model at $p \leq 0.06$ .This is due to higher probability of acceptance in the modified which resulted to less inspection of the lot. However at $p \geq 0.07$ the ATI of the optimal RDS plan in the modified model became higher than the ATI of the RDS plan in the existing model. This is because of higher probability of rejection of the lot at $p \geq 0.07$ than the existing model hence the increase in ATI.

Therefore optimal RDS plan in the modified model with ATI value of 113.69 is more economical than the optimal RDS plan with ATI of 237.32 in the existing model.

Table 9: Comparison of the Total Cost in the optimal RDS plans of the existing model and the modified model

	RDS plan (existing model)					RDS plan(modified model)					
p	n_1	c_1	n_2	c_2	TC	n_1	c_1	n_2	c_2	TC	Difference in TC
0.01	96	3	192	11	209.25	10	2	20	10	130.60	37.59%
0.02	96	3	192	11	342.83	10	2	20	10	259.86	24.20%
0.03	96	3	192	11	534.60	10	2	20	10	448.45	16.11%
0.04	96	3	192	11	781.18	10	2	20	10	720.69	7.74%
0.05	96	3	192	11	996.56	10	2	20	10	979.15	1.75%
0.06	96	3	192	11	1139.58	10	2	20	10	1153.03	-1.18%
0.07	96	3	192	11	1228.56	10	2	20	10	1254.74	-2.13%
0.08	96	3	192	11	1289.28	10	2	20	10	1319.07	-2.31%
0.09	96	3	192	11	1336.95	10	2	20	10	1368.51	-2.36%
0.1	96	3	192	11	1378.86	10	2	20	10	1412.65	-2.45%

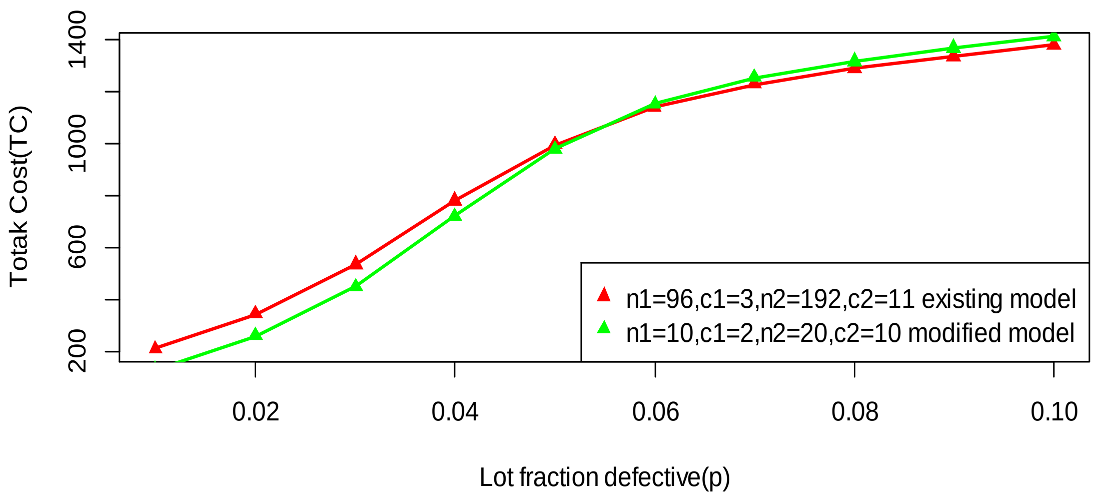

Fig.7:Total Cost(TC) in optimal RDS plans under the existing model and the modified model

Tables 5 and 6 above show high Total Cost (TC) of 534.43 and low Total Cost (TC) of 448.45 respectively for optimal RDS plan in the existing model and the modified model. The Total Cost (TC) for all the sampling plans in the two models increased as the fraction defective (p) increased as stated in table 9 and fig.7 above.The Total Cost (TC) in the modified model is lower than the total cost in the existing model at $p \leq 0.06$ .However at $p \geq 0.07$ the total cost in the modified model became higher than the total cost in the optimal RDS plan in the existing model. This is because at $p \leq 0.06$ the probability of acceptance is higher in the modified model resulting to low inspection cost. However, the probability of acceptance decreased at $p \geq 0.07$ leading to 100% inspection of the rejected lots hence the increase in total cost. Therefore since the total cost in the modified model for RDS is lower than in the existing model, the modified model is more economical than the existing model.

CONCLUSION

An economic single sampling plan developed by Hsu and Hsu (2012) is modified to incorporate inspection error and extended to rectifying double sampling plan. Optimal sampling plans that minimize the total cost incurred during inspection and satisfied both the producer's and consumer's risk requirement is obtained. Comparison between the existing model and the modified model is made. It is found that the modified model performed better and is more economical than the existing model in terms of cost. Furthermore the modified model provided more protection for both the producer and the consumer with low producer's risk (α) $=1 - P_a(AQL)$ and consumer's risk $(\beta) = P_{a_e}(LTPD_e)$ of 0.76% and 8% respectively while the producer's risk (α) $=1 - P_a(AQL)$ and consumer's risk $(\beta) = P_{a_e}(LTPD_e)$ in the existing model is 0.93% and 9.71% respectively.

REFERENCES

Amitava, M. (2016). *Fundamentals of Quality Control and Improvement*. 4th Edition, John willey & Sons,Inc,New Jersey, Canada, Pp.502-510

Anuja, S., Ratika, S., Tulka, S., and Singh, V. (2013). A Novel Method for Dynamic Sampling Plan and Inspection Policies for Quality Assurance. *Asian Research Journal of Business Management*, 2(1):34-43

Banovac, E., Darko, P., and Nikola, V. (2012). Mathematical Aspect of Acceptance Sampling Procedure. *International Journal of Mathematical Models and Methods in Applied Sciences,* 5(6):1-9

Castillo –villar, K. K.., Smith, N. R.., and Simonton, J. L. (2012). The Impact of the Cost of Quality on Serial-chain Network Design. *International Journal of Production Research,* 50(19): 5544-5566.

Chen, C., Espanchin-Niell, R. S., and Haight, G. R. (2016). Optimal Inspection of Imports to prevent Invasive Pest introduction.pp.1-40

Ching-Ho,Y., Heng, M., Chi-Huang, Y., and Chia-Hao, C. (2015).The Economic Design of Variable Acceptance Sampling Plan with Rectifying Inspection. *Kybernetes,* 44(3):440-450.

Ezzatallah, B., and Bahram, S. (2011). Chain Sampling Plan using Fuzzy Probability Theory. *Journal of Applied Sciences.* 11(24): 3830-3838.

Fallahnezhad, M. S., Sajjadieh M., Abdollahi, P. (2014). An Iterative Decision Rule to Minimize Cost of Acceptance Sampling Plan in Machine Replacement Problem. *International Journal of Engineering,* 27(7): 1099-1106.

Fallahnezhad, M. S., Ahmadi, Y. (2016). A New Optimization Model for Designing Acceptance Sampling plan Based on Run Length of conforming units. *Journal of Industrial and System Engineering,* 9(2): 67-87.

Fallahnehad, M. S., and Qazvini, E. (2016). A new Economical Scheme of Acceptance Sampling plan in a two-stage approach based on the Maxima Nomination Sampling Technique. Publish online in Transactions of the Institute of Measurement and Control.

Fallahnezhad,M,Qazvini,E and Abessi, M. (2017).Designing and Economical Acceptance Sampling plan in the presence of inspection errors based on Maxima Nomination Sampling Method. *Scientia Iranica,* 25(3): 1701-1711.

Frishman, F. (1960). An extended Chain Sampling Inspection Plan. *Journal of Industrial Quality Control,* 17: 10-12.

Gao, F. (2003). Studies on Chain Sampling Schemes in Quality and Reliability Engineering. A Thesis Submitted for the Degree of Doctor of Philosophy Department of System and Industrial Engineering. National University of Singapore.

Hagenimana et al. (2016). Technical Application for Inspection Sampling for Repairable

Systems in an Economic System. SpringPlus.Open Acess.pp.1-20.

Hoang, P. (2006). *Springer Handbook of Engineering Statistics*. Springer-verlag London Limited, Pp.264.

Hsu, L., and Hsu, J. (2012). Economic Design of Acceptance Sampling plans in a Two-Stage Supply Chain. *Advances in decision Science,* 2012: 1-14.

Jalbout,A Hadi,S Alkahby,Y , Fouad N. Jalbout Abdulla Darwish (2002). The Effects of Inspection Errors on the Average Outgoing Quality in an Industrial Process. *Electronic Journal of Mathematical and Physical Sciences.vol.1, pp.15-21*

Mistra, R. C., and Sandilya, A. (2009). *Reliability and Quality management*. New Age International Publishers, Ansari Road, Daryaganj, New Delhi, Pp.132-133.

Montgomery. D., and Jenings, C. (2011). *Managing, Controlling and Improving Quality*. New Jersey: John Willey and Sons, Pp.632-636.

Montgomery, D. C. (2009). *Introduction to Statistical Quality Control*. 6th Edition, John Willey & Sons, Inc.Arizona USA, Pp. 632-650.

Muhammad, B., and Chang, W. (2016). Minimization of Inspection Cost by Determining the Optimal Number of Quality Inspectors in the Garment Industry. *Indian Journal of fibre and textile research, 41*: 346-350.

Muhammad, A. S., Randolph, K. Henriqueta, N., and Antonio, A. (2017). Cost of Quality: Evaluating Cost –Quality Trade-offs for Inspections Strategies of Manufacturing Processes. *International Journal of Production Economics*, 188: 156-166.

Namin, S., Pakzad, A., and Nezhad, M. (2017). A DEA –Bases Approach for Multi-Objective Design of Attribute Acceptance Sampling Plans. *International Journal of Data Envelopment Analysis,* 5(2): 12-24.

Nezhad, M. S., and Nasab, H. H. (2011). Designing a Single Stage Acceptance Sampling Plan Based on the Control Threshold Policy. *International Journal of Industrial and Production Research*, 22(3): 143-150.

Nirma, M., Sabramania, R., and Uma, G. (2016). Selection of Optimum Single Sampling Plans under Prior Binomial Distribution by Minimizing the Average Acceptance Cost. *International Journal of Scientific and Research Publications*, 6(9):325-338

Subramaniam, S., and Yuvaraj, V. (2011). Bayesian Single Sampling Plan fro Attributes based on a Two point Prior Distribution. *International Journal of Civil Engineering Research*, 2 (2): 283-290.

Valtteri, T. (2012). Cost Modelling of Inspection Strategies in Automotive Quality Control. *Journal of Engineering Management Research,* vol.1(2): 33-38

Yinfeng , G, , and Loon, C. T.,(2006).Chain Sampling Scheme under Constant Inspection Errors.*Quality and Reliability Engineering Journal.Vol.22, pp.889-903.*