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Economic Design of Rectifying Single Sampling Plan (RSS) when Inspection Error is considered

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ABSTRACT

In this paper, Hsu and Hsu (2012) Single-sampling cost minimization model is modified to incorporate inspection error. The modified model aims to maximize the probability of lot acceptance with acceptable quality level (AQL) and minimizes the probability of lot acceptance with lot tolerant percent defective (LTPD). Optimal sampling plan that minimize the total cost and satisfied both the producer's and the consumer's risk requirement is determined. Comparison between the existing model and the modified model using the Operating characteristic (OC) curve, Average Total Inspection (ATI) and Total Cost (TC) is made. Sensitivity analysis is also carried out. Results showed that the modified model performed better and is more economical than the existing model in terms of cost effectiveness.

Keywords: Acceptable Quality Level, Lot Tolerant Percent defective Average Total Inspection, producer's risk, Consumer's risk.

INTRODUCTION

Acceptance Sampling is a method in statistical quality control used by inspectors for lot sentencing where a sample is taken from a lot and a decision to accept or reject the lot is taken based on information from the sample. According to Mishra and Sandilya (2009), Acceptance Sampling is a process of evaluating a portion of the product in a lot for the purpose of accepting or rejecting the lot on the basis of conforming or not conforming to quality specifications. According to Amitava (2016), Acceptance sampling can be performed during inspection of incoming raw materials, components, and assemblies, in various phases of in-process operations, or during final product or service inspection. It can be used as a form of product inspection between companies and their vendors, between manufacturers and their customers. Acceptance sampling plan is classified either by variable or attributes. In acceptance sampling by variable quality characteristics is measured using numerical value while in acceptance sampling plan by attributes, quality characteristics that are expressed on a "go, no-go" basis (Montgomery 2009).

Single sampling is one of the acceptance sampling by attribute where a single sample is taken from a lot and based on the single sample the lot is either accepted or rejected. The operating characteristic (OC) curve is therefore used for evaluating the performance of an acceptance sampling plan. Two levels of quality called Acceptable quality level (AQL) and Lot Tolerant Percent Defective (LTPD) are considered on the OC curve. AQL represents the quality level required by the consumer. Thus consumers want defect level equal to or better than the AQL. On the other hand LTPD (Lot Tolerant percent Defective) is the poorest level of quality that the consumer is willing to accept in an individual lot (Montgomery, 2009). The consumer demands the sampling plan to have a low probability of lot acceptance with a defect level as high as the LTPD.

Sampling inspection is never 100% reliable and involves two types of errors known as type I and Type II inspection errors. Type I inspection error occurs when non-defective unit is misclassified as defective, while type II inspection error occurs when a defective units is misclassified as non-defective unit. The probability of rejecting a lot with acceptable quality level (AQL) due type I error is known as producer's risk. The probability of accepting a lot with quality level equal or higher than LTPD is known as consumer's risk.

Nezhad and Nasab (2011) developed a cost model for acceptance sampling problem with objective of finding a constant control level that minimizes the total cost of rejection, cost of inspection and the cost of accepting defective units. Subramaniam and Yuvara (2011) proposed a model to minimize average cost over different combinations of Bayesian acceptance sampling plans with regards to the cost of accepting defective units in order to achieve optimal Bayesian Single Sampling Plan for attribute. Valtteri (2012) developed a cost model for comparing different inspection strategies and for creating an understanding of the structure of the costs of bad quality in automotive manufacturing. The model points out the fact that choosing a correct inspection strategy for quality control will lead to a significant increase in the profit margin of the business. Castillo-Villar *et al.* (2012) developed a model for supply-chain design that considers the Cost of Quality as well as the traditional manufacturing and distribution costs (SC-COQ model. Anuja *et al.* (2013) provided a cost effective solution for appropriate sampling plan to minimize cost of inspection and maintain quality of product using novel ABCDE classification of product so that different categories of products follow optimum plan of sampling. Fallahnezhad *et al.* (2014) presented an optimal iterative decision rule for minimizing total cost in designing a sampling plan for machine replacement problem using the approach of dynamic programming. Ching-ho *et al*. (2015) developed an economic cost model for variable acceptance sampling that minimizes quality cost in lots manufacturing. Muhammad and Chang (2016) developed a model to reduce inspection cost in acceptance single sampling plan by determining the optimal number of quality inspectors with respect to their skill levels using goal programming. Hagenimana *et al.* (2016) developed a model for determining the appropriate level of inspection sampling for manufacturers which considers the interest of the consumers who wish to minimize cost of production while ensuring that the final product is of high quality. Chen *et al*. (2016) developed a model with optimal sampling strategy that can be used in acceptance single sampling to reduce the introduction of damaging pests on agricultural imports. Nirmala *et al*. (2016) developed a model for finding optimum single sampling plans based on prior binomial distribution by minimizing the average acceptance cost such that the cost of accepting a defective unit is less and both producer's and consumer's risks are minimized. Fallahnezhad and Ahmadi (2016) presented an optimization model for designing an acceptance sampling plan based on cumulative sum of run length of conforming units. The objective was to minimize the total losses for both the producer and the consumer. Fallahnezhad and Qazvini (2016) presented a new economical scheme of the acceptance sampling plan in a two-stage approach based on the Maxima Nomination Sampling technique. Muhammad *et al*. (2017) developed a cost model for the evaluation of inspection strategies in manufacturing system to minimize cost and maximize quality of products. Namin (2017) extended the model of Lie-fern and Jia-Tzer (2012) to a multi-objective economic-statistical design (MOESD) for acceptance single sampling plan to strike a balance between cost and quality features

In this paper, Hsu and Hsu (2012) economic single-sampling plan model which assumed "perfect" inspection is modified to incorporate inspection error. The modified model maximizes the probability of lot acceptance with acceptable quality level (AQL) and minimizes the probability of lot acceptance with lot tolerant percent defective (LTPD). Optimal sampling plan to optimize the total cost is determined and comparison of the performances between the existing model and the modified model is made.

MATERIALS AND METHODS

Inspection Errors and Producer's and Consumer's Risks

Inspection error occurs if non-defective unit is misclassified as defective or a defective unit is misclassified as non-defective. An outline of the derivation as given by Gao (2003) is as follows:

Let T (true) and A (apparent) represent the true and the observed units respectively.

Define:

T= 0 when the inspected unit is truly non-defective.

T= 1 when the inspected unit is truly defective, and

A=0 when the inspected unit is observed (or classified) as non-defective.

A=1 when the inspected unit is observed (or classified) as defective.

The following table gives all possible combinations of the realization of the two random variables and the two types of errors:

Table 1: Types of inspection error

Inspection result	T = 0	T= 1
A= 0	No risk	Type ll error (e_2)
A = 1	Type l error (e_1)	No risk

In a similar way, the observed fraction defective p_e is defined as the probability Pr(A=1) that a randomly selected unit is classified 'defective' as shown on the table below:

Table 2: Probability of the realisation of two state variables

State variable	Realisation	Probability of occurrence	Interpretation of the probabilities
T	1	p	True fraction defective
	0	$1-p$	True proportion of non-defective units
A	1	p_e	Observed fraction 'defective'
	0	$1-p_e$	Observed proportion of 'non-defective' units

Apparent (Observed) Fraction defective p_e is the fraction of incoming lot which is observed as defective by the inspector. Let p be the true fraction defective and is written as:

$$p_e = p(1-e_2) + (1-p)e_1 \quad (1)$$

Acceptance Single Sampling plan

A random sample of size n is randomly taken from a lot size of N, the number of defective units in the sample x is compared with the acceptance number c.If $x \leq c$ the lot is accepted. However, the lot is rejected if $x > c$.

Probability of acceptance P_a for acceptance single sampling plan under error-free inspection assumption using binomial distribution is as given:

$$P_a = p(x \leq c) = \sum_{x=0}^{c} \binom{n}{x} p^x (1-p)^{n-x} \quad (2)$$

Where c =the acceptance number, n = sample size and p = the true fraction defective units

When inspection error is taken into consideration, the probability of acceptance causes the value of the true fraction defective p in eqn (2) to be replaced by the value of the apparent fraction defective (p_e) in equation (1) .Thus equation (2) becomes

$P_{a_e} = \sum_{x=0}^{C} \binom{n}{x} [e_1(1-p) + p(1-e_2)]^x . [1 - e_1(1-p) - p(1-e_2)]^{n-x}$,

$P_{a_e} = \sum_{x=0}^{C} \binom{n}{x} {p_e}^x (1-p_e)^{n-x}$ (3)

Rectifying Single Sampling (RSS) Plan

When rectifying inspection is carried out on acceptance single sampling plan, all the defective units in the sample and in the rejected lot is subjected to 100% inspection where all the observed defective units are replaced with non-defective units.

Defective Units in Rectifying Single Sampling (RSS) Plan

Incoming lots to the inspection activity have fraction defective units. Some of these lots are accepted, and others are rejected. The rejected lots are screened, and all the defective units are detected and removed. However, accepted lots have fraction defective units which are not detected. Hsu and Hsu (2012) denoted proportion of defective unit not detected as (Dn) and proportion of defective units detected as (Dd) respectively as stated below:

$Dn = (N-n)pP_a$ (4), $Dd = np - p(1 - P_a)(N-n)$ (5)

When inspection error is considered, defective unit not detected (Dn_e) and proportion of defective units detected (Dd_e) is stated below:

$Dn_e = npe_2 + p(N-n)P_{a_e} + p(N-n)(1 - P_{a_e})e_2$ (6)

$Dd_e = np(1-e_2) + p(N-n)(1-e_2)(1-P_{a_e})$ (7)

Average Total Inspection (ATI)

Average total inspection is another measure of rectifying inspection. It consists of the average number of units inspected per lot. The inspected units could be in the sample as well as in the rejected lot given that the lot quality is $0 < p < 1$. Average Total Inspection (ATI) in RSS plan with error-free inspection error is given as:

$ATI = n + (1 - P_a)(N-n)$ (8)

When inspection error is considered eqn (8) is rewritten as: $ATI_e = n + (1 - P_{ae})(N-n)$ (9)

Probability of detecting defective unit in a sample

If one or more defective units are detected in the sample, the probability of observing one or more defective units in the sample is:

$P(x \geq 1) = 1 - P(x=0) = 1 - (1-p)^n$ (10)

Lot proportion defective according to Fallahnezhad *et al*. (2018) is then given as:

$p = 1 - (1-p)^n$ (11)

Apparent Proportion Defective (p_e)

Apparent fraction defective p_e is thus obtained as:

$p_e = (1-e_2)p + (1-p)e_1$ (12)

$p_e = 1 - (1-p)^n(1-e_2) + e_1(1-p)^n$ (13)

Determination of Producer's risk and Consumer's risk in the Design of Single Sampling Plan with AQL and LTPD quality level

Consider the producer's risk (α) and its associated quality level$p_1 = AQL$ as well as the consumer's risk (β) with its associated quality level $p_2 = LTPD$,we formulate the probabilities of lot acceptance $1-\alpha$ with quality $p_1 = AQL$ and β with quality levels $p_2 = LTPD$ for single sampling plan under error-free and with inspection error assumptions:

Probability of lot acceptance at $p_1 = AQL$ is formulated below:

$1 - \alpha = P_a(x \leq c|n, p_1 = AQL) = \sum_{x=0}^{C} \binom{n}{x} {p_1}^x (1-p_1)^{n-x}$(3.43) (14)

$1 - \alpha = P_a(AQL)$ (15)

The probability of rejection at $p_1 = AQL$ or producer's risk (α) is:

$1 - P_a(AQL) = \alpha$ (16)

When inspection error is considered, AQL and LTPD are replaced with apparent acceptable quality level (AQLe) and apparent Lot Tolerant Percent defective (LTPDe) respectively.We then formulate probability of acceptance $1-\alpha$ with inspection error for lot with quality level $p_1 = AQL_e$ as:

$$1-\alpha = P_{ae}(x \le c|n, p_1 = AQL_e) = \sum_{x=0}^{C} \binom{n}{x} AQL_e{}^x (1-AQL_e)^{n-x} \quad (17)$$

According to Banovac et al (2012)

$$AQL_e = AQL(1-e_2) + e_1(1-AQL) \quad (18)$$

$$AQL_e = 1-(1-AQL)^n(1-e_2) + e_1(1-AQL)^n \quad (19)$$

Substituting (19) for AQLe in (17) we have:

$$= \sum_{x=0}^{c} \binom{n}{x} \{1-(1-AQL)^n(1-e_2) + e_1(1-AQL)^n\}^x \{(1-AQL)^n(1-e_2) + e_1(1-AQL)^n\}^{n-x} \quad (20)$$

The probability of lot rejection at $p_1 = AQLe$ (producer's risk) in is thus:

$$1 - P_{ae}(AQL_e) = \alpha \quad (21)$$

The probability of accepting lot with quality level $p_2 = LTPD$ or consumer's risk under error-free inspection is given as:

$$\beta = P(x \le c|n.p_2 = LTPD) = \sum_{x=0}^{C} \binom{n}{x} p_2{}^x (1-p_2)^{n-x} \quad (22)$$

$$\beta = \sum_{x=0}^{c} \binom{n}{x} LTPD^X (1-LTPD)^{n-x}$$
(23)

$$\beta = P_a(LTPD) \quad (24)$$

When inspection error is considered the probability of acceptance at $p_2 = LTPD$ is obtained by replacing $p_2 = LTPD$ with $LTPD_e$.Thus the probability of acceptance is given as

$$\beta = P(x \le c|n.p_2 = LTPD_e) = \sum_{x=0}^{C} \binom{n}{x} LTPD_e{}^x (1-LTPD_e)^{n-x} \quad (25)$$

$$\beta = P_{ae}(x \le c|n, p_2 = LTPD_e) = \sum_{x=0}^{c} \binom{n}{x} [\{1-(1-LTPD)^n\}(1-e_2) + e_1(1-LTPD)^n]^x [\{1-[(1-LTPD)^n]\}(1-e_2) + e_1(1-LTPD)^n]^{n-x}$$
(26)

$$\beta = P_{ae}(LTPD_e) \quad (27)$$

Cost Minimization Model,

Hsu and Hsu (2012) single sampling cost model as presented below:

$$\text{Minimize Total cost } (TC) = C_i ATI + C_f Dd + C_o Dn \quad (28)$$

Subject to $1 - P_a(AQL) \le \alpha$

$P_a(LTPD) \le \beta$

Where TC is the total cost, C_i is the cost of inspection per unit,C_f is the internal failure cost (which include cost of repair, scrap or rework of defective unit) and C_o is the external failure cost or post sales cost (which include repair or replacement cost). α and β represent producer's and consumer's risk respectively.

Modified Cost Minimization Model

Hsu and Hsu (2012) acceptance single sampling model is modified to incorporate inspection error. Additional objective functions to maximise the probability of acceptance at given acceptable quality level and minimize the probability of acceptance at given lot tolerant percent defective are introduced. The modified model is as presented below:

$$\text{Minimize Total Cost } (TC) = C_i ATI_e + C_f Dd_e + C_o Dn_e \quad (29)$$

Maximize $P_{a_e}(AQL_e)$

Minimize $P_{a_e}(LTPD_e)$

Subject to $1 - P_{a_e}(AQL_e) \le \alpha$ $(LTPD_e) \le \beta$

AQL_e and $LTPD_e$ represent apparent (observed) Acceptable Quality Level and apparent (observed) Lot Tolerant Percent Defective respectively. Other parameters with inspection error are as stated above.

Algorithm to determine optimal sampling plan in RSS.

The algorithm to determine optimal sampling plan using the modified model is as stated below, though the same approach can be applied in existing model.

Step1: Specify the parameter values α=0.05, β=0.1, $e_1 = 0.01, e_{2,} = 0.01, p = 0.03, AQL = 0.02, LTPD = 0.07, C_{i,} = 1, C_{f,} = 2$,and $C_o = 10$ in the model

Step 2: Set the values as $n \le 205$ and$C = \{1, \dots, 15\}$.

Step3: Using the values in step 1 and 2 above, obtain the acceptance probabilities of the lot AQL_e and $LTPD_e$ i.e, $P_{ae}(AQL_e)$ and $P_{ae}(LTPD_e)$
Step 4: Select the parameters where the constraints $1 - P_{ae}(AQL_e) \leq \alpha$ and $P_{ae}(LTPD_e) \leq \beta$ are simultaneously satisfied.
Step 5: Calculate the cost function in each case using the cost model.
Step 6: Continue the process with different combinations of n and c until the minimum cost is obtained.
Step 7: Select the sampling plan with the smallest sample size and minimum cost as the optimal solution

RESULTS AND DISCUSSION

Determination of optimal Sampling Plans for RSS plan in the existing model and the modified model.

Table 3: Rectifying Single Sampling (RSS) Plans with error-free inspection assumption satisfying the parameters AQL=0.02, LTPD=0.07, α=0.05, β=0.1, p=0.03 with$n \leq 250$

n	c	AOQ	ATI	D_n	D_d	$1 - P_a(AQL)$	$P_a(LTPD)$	$P_a(p)$	TC
196	7	0.0184	386.83	18.40	11.60	0.0448	0.0322	0.7627	593.99
196	8	0.0208	306.47	20.81	9.19	0.0180	0.0642	0.8626	532.92
197	7	0.0183	390.88	18.27	11.73	0.0459	0.0309	0.7586	597.07
197	8	0.0207	309.74	20.71	9.29	0.0185	0.0620	0.8596	535.41
198	7	0.0182	394.95	18.15	11.85	0.0470	0.0297	0.7544	600.16
198	8	0.0206	313.04	20.61	9.39	0.0190	0.0598	0.8566	537.91
199	7	0.0180	399.03	18.03	11.97	0.0482	0.0285	0.7503	603.26
199	8	0.0205	316.35	20.51	9.49	0.0196	0.0577	0.8535	540.42
200	7	0.0179	403.12	17.91	12,09	0.0493	0.0274	0.7461	606.37
200	8	0.0204	319.68	20.41	9.59	0.0202	0.0556	0.8504	542.95
201	8	0.0203	323.03	20.31	9.69	0.0208	0.0537	0.8473	545.50
201	**9**	**0.0220**	**267.19**	**21.98**	**8.02**	**0.0077**	**0.0979**	**0.9172**	**503.07**
202	8	0.0202	326.40	20.21	9.79	0.0214	0.0518	0.8441	548.06
202	9	0.0219	269.78	21.91	8.09	0.0080	0.0947	0.9151	505.03
203	8	0.0201	329.79	20.11	9.89	0.0220	0.0499	0.8409	550.64
203	9	0.0218	272.39	21.83	8.17	0.0083	0.0917	0.9129	507.02
204	8	0.0200	333.19	20.00	10.00	0.0226	0.0481	0.8377	553.23
204	9	0.0217	275.03	21.75	8.25	0.0085	0.0888	0.9108	509.02
205	8	0.0199	336.62	19.90	10.10	0.0232	0.0464	0.8344	555.83
205	9	0.0217	277.68	21.67	8.33	0.0088	0.0859	0.9086	511.04

Table 4: Rectifying Single Sampling (RSS) Plans with inspection error satisfying the parameters AQL=0.02, LTPD=0.07, α=0.05, β=0.1, p=0.03, with $n \leq 250$

n	c	AOQ_e	ATI_e	D_{ne}	D_{de}	$1 - P_{ae}(AQL_e)$	$P_{ae}(LTPD_e)$	$P_{ae}(p)$	TC
13	5	0.0237	212.67	23.68	6.32	0.0471	0.0967	0.7977	462.14
16	7	0.0231	231.49	23.12	6.88	0.0388	0.0412	0.7810	476.49
17	8	0.0241	199.03	24.09	5.91	0.0265	0.0413	0.8148	451.74
18	9	0.0248	177.35	24.82	5.18	0.0185	0.0402	0.8404	432.92
19	9	0.0217	279.32	21.70	8.30	0.0395	0.0125	0.7346	512.96
19	10	0.0254	155.50	25.38	4.62	0.0133	0.0382	0.8609	418.55
19	11	0.0276	79.26	27.65	2.35	0.0037	0.0974	0.9386	360.43
20	10	0.0225	253.65	22.47	7.53	0.0295	0.0117	0.7616	493.38
20	11	0.0258	141.07	25.81	4.19	0.0097	0.0356	0.8765	407.55
20	12	0.0278	73.04	27.83	2.17	0.0027	0.0910	0.9459	355.69
21	11	0.0231	233.27	23.07	6.93	0.0225	0.0107	0.7832	477.84
21	12	0.0261	130.03	26.14	3.86	0.0073	0.0326	0.8886	399.13
21	13	0.0280	68.52	27.96	2.04	0.0020	0.0838	0.9515	352.24
22	11	0.0195	352.56	19.53	10.47	0.0462	0.0027	0.6620	568.79
22	12	0.0236	217.12	23.55	6.45	0.0174	0.0096	0.8005	465.53
22	13	0.0264	121.59	26.39	3.61	0.0056	0.0293	0.8982	392.70
22	14	0.0281	65.28	28.06	1.94	0.0015	0.0763	0.9557	349.77
23	12	0.0201	332.46	20.13	9.87	0.0371	0.0023	0.6833	553.47
23	13	0.0239	204.40	23.83	6.07	0.0138	0.0084	0.8143	455.84
23	14	0.0266	115.21	26.58	3.42	0.0044	0.0261	0.9056	387.83
23	**15**	**0.0281**	**63.02**	**28.13**	**1.87**	**0.0012**	**0.0687**	**0.9590**	**348.05**
24	13	0.0206	316.24	20.61	9.39	0.0302	0.0020	0.7006	541.10
24	14	0.0242	194.47	24.22	5.78	0.0111	0.0073	0.8253	448.26
24	15	0.0267	110.45	26.72	3.28	0.0035	0.0228	0.9114	384.21
25	14	0.0210	303.28	20.99	9.01	0.0249	0.0017	0.7146	531.22
25	15	0.0245	186.82	24.45	5.55	0.0091	0.0062	0.8341	442.43
26	15	0.0213	293.11	21.29	8.71	0.0209	0.0014	0.7258	523.46
27	15	0.0175	422.00	17.47	12.53	0.0431	0.0003	0.5940	621.73

Tables 3 and 4 represent the sampling plans generated which satisfies the conditions in the two models stated above. The existing model show optimal sampling plan of $n = 201, C = 9$ with minimum total cost of 503.07. In the modified model the optimal sampling plan is $n = 23, C = 15$ with minimum total cost of 348.05. Table 4 shows minimum values for sample size(n), the ATI, producer's risk$(1 - P_{ae}(AQLe))$, consumer's risk $\left(P_{ae}(LTPDe)\right)$ and the Total Cost (TC) in the optimal sampling plan of the modified model than the optimal values in the existing model. This suggests that the modified model is more economical than the existing model.

Table 5: Comparison of the probability of acceptance of optimal sampling plans of RSS plans under the existing model and the modified model

$\alpha = 0.05$		β= 0.1	
Existing model optimal sampling plan: $n = 201, c = 9$		modified model optimal sampling plan: $n = 23, c = 15$	
p	P_a(p)	p	Pa_e(p)
0.01	1.0000	0.01	1.0000
0.02(AQL)	0.9923=$P_a(AQL)$	0.02	0.9988=$P_{ae}(AQL_e)$
0.03	0.9172	0.03	0.9590
0.04	0.7141	0.04	0.7659
0.05	0.4483	0.05	0.4534
0.06	0.2290	0.06	0.1994
0.07(LTPD)	0.0978=$P_a(LTPD)$	0.07	0.0687=$P_{ae}(LTPD_e)$
0.08	0.0358	0.08	0.0197
0.09	0.0115	0.09	0.0049
0.1	0.0033	0.1	0.0011

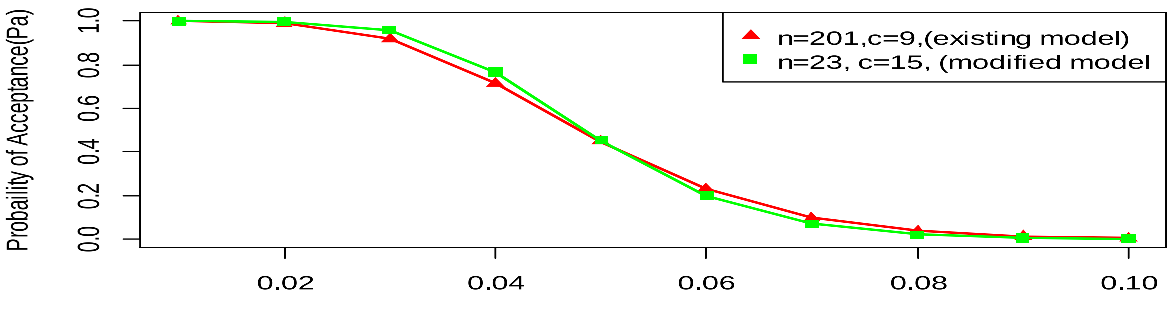

Fig.1:OC curves for Rectifying Single Sampling (RSS) Plans

Figure1 above is the operating characteristics curve for RSS plan under the existing model and the modified model. Generally, the probability of acceptance for all the sampling plans decreased as the fraction defective units of the lot increased. All the models show high probability of acceptance at $AQL \leq 0.02$ and low probability of acceptance at $LTPD \geq 0.07$.However the probability of acceptance of the modified model is higher than that of the existing model $AQL \leq 0.02$ and lower than the existing model at $LTPD \geq 0.07$.The producer's risk (α) and the consumer's risk(β) in optimal sampling plan of the modified model is 0.0012 or 0.12% and 0.0687 or 6.87% respectively which is lower than the producer's risk (α) of 0.0077 or 0.77% and consumer's risk (β) of 0.0978 or 9.78% respectively in the optimal sampling plan of the existing model. It can be observed that the percentage of producer's risk and consumer's risk for optimal RSS plan in the modified model is lower than the percentage of producer's risk and consumer's risk in the RSS plan using the existing model. This showed that the modified model performed better and is more discriminatory than the existing model

.

Table 6: Average Total Inspection (ATI) of optimal RSS plans under the existing model and the modified model

optimal RSS plan(existing model): $n = 201, c = 9$		optimal RSS plan(modified model): $n = 23, c = 15$	
p	ATI	p	ATI
0.01	201.03	0.01	23.00
0.02	207.18	0.02	24.14
0.03	267.19	0.03	63.02
0.04	429.41	0.04	251.71
0.05	641.79	0.05	557.07
0.06	816.99	0.06	805.21
0.07	921.86	0.07	932.84
0.08	971.36	0.08	980.76
0.09	990.79	0.09	995.19
0.1	997.35	0.1	998.90

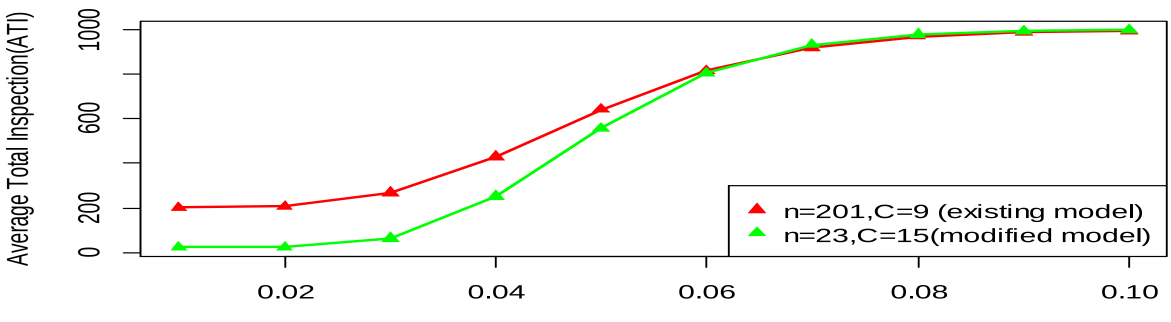

Fig.2:Average Total Inspection (ATI) for Rectifying Single Sampling (RSS) plans

RSS plans in tables 3 and 4 above show high ATI of 267.19 in the existing model and low ATI of 63.02 in the modified model. The Average Total Inspection (ATI) in all the sampling plans in the two models increased as the fraction defective (p) increased as shown in table 6 and figure 2 above. From table 6 and figure 2 above the Average Total Inspection (ATI) for the two models generally increase as the lot fraction defective(p) units increased. The Average Total Inspection (ATI) in the modified model is lower than that of the existing model at $AQL \leq 0.02$ and higher than that of the existing model at $LTPD \geq 0.07$.The lower ATI of the modified model at $AQL \leq 0.02$ is due to higher probability of acceptance which resulted to less inspection of the lot. The ATI in the modified model is also higher than that of the existing model at $LTPD \geq 0.07$ quality level. This is as a result of 100% inspection of the rejected lot which is caused by increased probability of lot rejection at $LTPD \geq 0.07$.

Table 7: Total Cost in the optimal RSS plans of the existing model and the modified model

	RSS plan(existing model)			RSS plan(modified model)			
p	n	c	TC	n	c	TC	Difference in TC
0.01	201	9	284.95	23	15	121.18	57.47%
0.02	201	9	374.03	23	15	220.31	41.09%
0.03	201	9	503.07	23	15	348.05	30.81%
0.04	201	9	692.00	23	15	571.92	17.35%
0.05	201	9	885.07	23	15	836.47	5.49%
0.06	201	9	1024.84	23	15	1022.57	0,22%
0.07	201	9	1105.62	23	15	1115.68	-0.91%
0.08	201	9	1149.69	23	15	1159.35	-0.84%
0.09	201	9	1177.42	23	15	1185.82	-0.71%
0.1	201	9	1199.47	23	15	1207.77	-0.69%

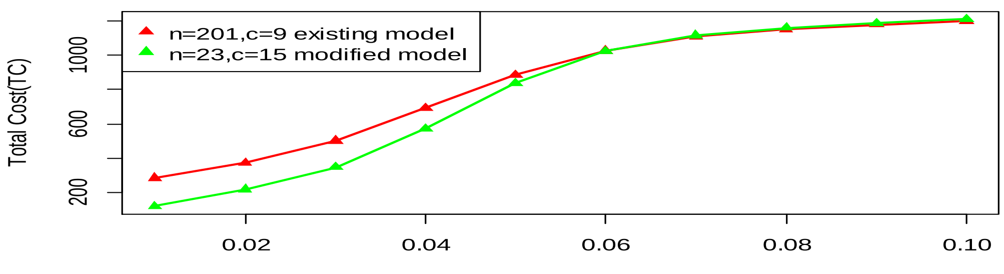

Fig.3:Total Cost(TC)for optimal Rectifying Single Sampling (RSS) plans

Tables 3 and 4 above show high Total Cost (TC) of 503.07 and low Total Cost (TC) of 348.05 respectively for optimum RSS plans in the existing model and the modified model. From table 7 and figure 3 above, it is also noted that the total cost generally increased in all the models as the fraction defective unit (p) increased. However, the total cost in the modified model is lower at $AQL \leq 0.02$ but higher than the existing model at $LTPD \geq 0.07$.This is because the probability of acceptance in the modified model is higher at $AQL \leq 0.02$ resulting to less inspection hence the decreased in total cost. On the other hand, the probability of rejection in the modified model is higher at $LTPD \geq 0.07$ than the existing model .Therefore 100% inspection is carried out on the rejected lot hence the higher the total cost.

Table 8: Effect of C_i,C_f and C_o on the TC in optimal RSS plans

parameters			Optimal RSS plan (existing model)			Optimal RSS plan (modified model)			
C_i	C_f	C_0	n	c	TC	n	c	TC	Difference in TC
0.5	2	10	201	9	369.47	23	15	316.54	14.33%
1.0	2	10	201	9	503.07	23	15	348.05	30.80%
1.5	2	10	201	9	636.66	23	15	379.56	40.38%
2.0	2	10	201	9	770.26	23	15	411.07	46.63%
2,5	2	10	201	9	903.85	23	15	442.53	51.04%
3.0	2	10	201	9	1037.45	23	15	474.09	54.30%
3.5	2	10	201	9	1171.05	23	15	505.60	56.83%
4.0	2	10	201	9	1304.64	23	15	537.11	58.83%
4.5	2	10	201	9	1438.24	23	15	568.62	60.46%
5.0	2	10	201	9	1571.84	23	15	600.13	61.80%
5.5	2	10	201	9	1705.43	23	15	631.64	62.96%
6.0	2	10	201	9	1839.03	23	15	663.15	63.94%
6.5	2	10	201	9	1972.62	23	15	694.66	64.78%
7.0	2	10	201	9	2106.22	23	15	726.17	65.52%
7.5	2	10	201	9	2239.82	23	15	757.68	66.17%
8.0	2	10	201	9	2373.41	23	15	789.19	66.75%
8.5	2	10	201	9	2507.01	23	15	820.70	67.26%
9.0	2	10	201	9	2640.60	23	15	852.21	67.73%
9.5	2	10	201	9	2724.20	23	15	883.72	67.56%
10.0	2	10	201	9	2907.80	23	15	915.24	68.52%
1	0.5	10	201	9	491.04	23	15	345.24	29.69%
1	1.0	10	201	9	495.05	23	15	346.18	30.07%
1	1.5	10	201	9	499.06	23	15	347.11	30.45%
1	2.0	10	201	9	503.07	23	15	348.05	30.82%
1	2.5	10	201	9	507.07	23	15	348.98	31.18%
1	3.0	10	201	9	511.08	23	15	349.92	31.53%
1	3.5	10	201	9	515.09	23	15	350.88	31.88%
1	4.0	10	201	9	519.10	23	15	351.79	32.23%
1	4.5	10	201	9	523.11	23	15	352.73	32.57%
1	5.0	10	201	9	527.11	23	15	353.66	32.91%
1	5.5	10	201	9	531.12	23	15	354.60	33.24%
1	6.0	10	201	9	535.13	23	15	355.53	33.56%
1	6.5	10	201	9	539.14	23	15	356.47	33.88%
1	7.0	10	201	9	543.14	23	15	357.41	34.20%
1	7.5	10	201	9	547.15	23	15	358.34	34.51%
1	8.0	10	201	9	551.16	23	15	359.28	34.81%
1	8.5	10	201	9	555.17	23	15	360.21	35.12%
1	9.0	10	201	9	559.18	23	15	361.15	35.41%
1	9.5	10	201	9	563.18	23	15	362.09	35.71%
1	10.0	10	201	9	567.19	23	15	363.02	35.99%
1	2	5	201	9	393.14	23	15	207.41	47.24%
1	2	10	201	9	503.07	23	15	348.05	30.81%
1	2	15	201	9	612.99	23	15	488.69	20.28%
1	2	20	201	9	722.91	23	15	629.33	12.95%
1	2	25	201	9	832.83	23	15	769.97	7.55%
1	2	30	201	9	942.75	23	15	910.61	3.41%
1	2	35	201	9	1052.67	23	15	1051.25	0.14%
1	2	40	201	9	1162.59	23	15	1191.90	-2.52%
1	2	45	201	9	1272.51	23	15	1332.54	-4.72%
1	2	50	201	9	1382.44	23	15	1473.18	-10.56%
1	2	55	201	9	1492.36	23	15	1613.82	-8.14%
1	2	60	201	9	1602.28	23	15	1754.46	-9.50%
1	2	65	201	9	1712.20	23	15	1895.10	-10.68%
1	2	70	201	9	1822.12	23	15	2035.74	-11.72%
1	2	75	201	9	1932.04	23	15	2176.39	-12.65%
1	2	80	201	9	2041.96	23	15	2317.03	-13.47%
1	2	85	201	9	2151.88	23	15	2457.67	-14.21%
1	2	90	201	9	2261.80	23	15	2598.31	-14.88%
1	2	95	201	9	2371.73	23	15	2738.85	-15.48%
1	2	100	201	9	2481.65	23	15	2879.59	-16.04%

Sensitivity Analysis

Table 8 above demonstrated the effect of increase in the inspection cost (C_i) , internal failure cost (C_f) and external cost (C_o) on the total cost for RSS plan in both the existing model and the modified model. It can be seen that in both the existing model and the modified model, the total cost increased as the inspection cost (C_i) also increased and other costs are constant. However, the total cost in the existing model is higher than the total cost in the modified model.

It is also observed that as the internal failure cost(C_f)increased and inspection cost (C_i) and external failure cost (C_o) are kept constant, the total cost in both the existing model and the modified model also increased. However the total cost using the existing model is significantly higher than the total cost in the modified model.

Similarly, as the external failure cost (C_o) increased and inspection cost (C_i) and internal failure cost(C_f) are kept constant, the total cost in both the existing model and the modified model also increased. However the total cost in the sampling plan using the existing model is significantly higher than the total cost in the modified model. It is to be noted however that when external failure cost increased to $(C_o) \geq 40$ the total cost in the modified model became higher than the total cost in the existing model.

It therefore means that the RSS plan in the modified model is better and more economical than the RSS plan in the existing model when $(C_o) \leq 35$.

CONCLUSION

In this paper an economic single sampling plan developed by Hsu and Hsu (2012) is adopted and modified to incorporate inspection error. Other conditions to maximize probability of acceptance at acceptable quality level (AQL) and minimize probability of acceptance at lot tolerant percent defective (LTPD) are added. Optimal sampling plan that minimize the total cost incurred during inspection and satisfied both the producer's and consumer's risk requirement is obtained. Comparison between the existing model and the modified model is made. The results showed that the modified model performed better and is more economical than the existing model. This study can be extended to other acceptance sampling plans such as double sampling plan, chain sampling plan and others sampling plans.

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