The calculation of Binding Energy and Incompressibility, Pressure, and Velocity of Sound of Infinite Nuclear matter Using New One Boson Interaction.

***Aper, M.T., Gbaorun, F., and Fiase, J.O.**
Department of Physics,
Benue State University, Makurdi,

*Corresponding Author: aperterwase@gmail.com

Abstract

The use of effective nucleon – nucleon (N N) interactions for the determination of nuclear matter properties such as, binding energy per nucleon, incompressibility,K_0 of infinite nuclear matter, pressure and velocity of sound of nuclear matter has been a subject of great interest to nuclear physicists for many decades. The effective interaction usually involved in these calculations has been the Michigan three Yukawa (M3Y) effective interactions whose origin is from G- matrix approach. In this research work however, we have used a newly developed interaction known as new one boson (NOB) effective interaction to carry out similar calculations. This new interaction is based on the Lowest Order Constrained Variational (LOCV) technique. The interaction reproduces the saturation energy of spin and isospin infinite nuclear matter of approximately -16MeV at the normal nuclear matter saturation density consistent with the best available density-dependent interaction derived from the G-matrix approach. The results of the incompressibility obtained using the NOB interaction ranges from 304 to 309 MeV. These values are in good agreement with the values of incompressibility obtained for similar calculations using the M3Y – Reid effective interaction, in which values for K_0 range from 304 to 310 MeV. The results of pressure and velocity of sound of infinite nuclear matter obtained in the present calculations are also in excellent agreement with results of other workers. The results of our present calculations indicate that, the NOB interaction has passed the basic test for an effective interaction. The NOB may therefore be applied to other nuclear matter and optical model calculations to ascertain its reliability.

Key words: Incompressibility, Infinite Nuclear Matter, One Boson Interaction, Angular momenta channels, Equation of State

Introduction

The study of nuclear matter properties using effective N- N interaction has received a lot of attention for some time now. A number of effective interactions have been developed by many authors to study the interaction between nucleons inside the nucleus (Bertsch *et al.*, 1977; Dartmans and Amos, 1994).

Among the different kinds of the effective interaction, the commonest is the so-called Michigan three Yukawa (M3Y) interactions which was first developed by Bertsch and collaborators (Bertsch *et al*., 1977). According to Misicu (2007), the M3Y interaction has been derived by fitting its matrix elements into an oscillator basis (the oscillator parameters were chosen to reproduce the ground state of ^{16}O) to the elements of the G–matrix obtained with the Reid soft-core N - N interaction. Due to the lengthy and cumbersome nature of the G – matrix calculations, Fiase *et al.* (2002), have derived a similarly motivated M3Y – type interaction which is however based on the Lowest Order Constrained Variational (LOCV) technique rather than the G – matrix approach. The results of some nuclear matter calculations carried out with the M3Y – type interaction compared favourably with the results obtained from that of the G – matrix calculations (Fiase *et al*., 2011).

The M3Y- type effective interaction has been found to be very successful in many nuclear matter calculations. Hosaka *et al* (1985) have however suggested that the simple local form of M3Y interaction proposed by Bertsch and collaborators need to be improved upon by taking more realistic functional form such as the One Boson Exchange Potential (OBEP) forms. With these new functional forms, they found an improvement over the formal interaction. Following the suggestion of Hosaka *et al.* (1985), Fiase *et al.* (2006), also derived a similarly effective One Boson Exchange Potential (OBEP) based on the lowest order constrained variation (LOCV) approach which promised to be very effective if its density –dependent factor is derived.

The aim of this work is to derive a density - dependent one boson effective interaction from the result of the work of Fiase *et al* (2006), and then use it to calculate the binding energy and compression modulus, K_0, pressure and velocity of sound of infinite nuclear matter.

Mathematical Formulation of the Effective One Boson Exchange Potential (OBEP)

The effective OBEP whose oscillator matrix elements fit the elements of an effective two–body interaction consists of three terms; the central potential term *Vc* ®, the spin-orbit potential term *Vis*® and the tensor potential term *Vtn*(*r*)and is given by (Fiase *et al*, 2006).

$$V_c(r) = \sum_j A_j Y_c({}^{r}\!/_{R_j})_c$$

$$V_{is}(r) = \sum_j A_j Y_{is}({}^{r}\!/_{R_j})\hat{L}\vec{S}$$

$$V_{tn}(r) = \sum_j A_j Y_{tn}({}^{r}\!/_{R_j})S_{ij} \qquad 1$$

where; $Y_c(x) = \frac{e^{-x}}{x}$,

$$Y_{is}(x) = (1+\tfrac{1}{x})\frac{e^{-x}}{x^2},$$

$$Y_{tn}(x) = (1+\tfrac{3}{x}+\tfrac{3}{x^2})\frac{e^{-x}}{x} \qquad 2$$

In equation (2), the A_j are the strengths of the interaction which are determined by fitting the oscillator matrix elements of (2) to our two – body matrix elements of equation (1), the R_j are ranges. The ranges for the central forces were chosen to be 0.20, 0.33, 0.50 and 1.414fm which were motivated by the one boson exchanges. The longest range corresponds to the one–pion exchange, while the shortest range corresponds to heavier mesons such as *σ, ρ* and *ω* mesons.

From the work of Fiase *et al*. (2006), the results of their calculation were separated into various angular momenta channels which were; the singlet-even (SE), singlet-odd (SO), triplet-even (TE), triplet-odd (TO), the spin-orbit and the tensor channels. Using the results of these angular momenta channels, we derive the density dependent effective interaction as follows:

The direct (V^D) and exchange (V^{EX}) terms of our NOB interaction in the singlet-even (SE), singlet–odd (SO), triplet–even (TE) and triplet–odd (TO) channels can be recast into spin and isospin formalism as (Chaudun, 1986):

$$V^D = \tfrac{1}{16}(3V^{SE} + 3V^{TE} + V^{SO} + 9V^{TO}), \qquad 3$$

with

$$V^{EX} = V^D P^\sigma P^\tau$$

where $_\sigma P$ and $_\tau P$ are the projections operators in the spin and isospin channels. Hence;

$$V^{EX} = \tfrac{1}{16}(3V^{SE} + 3V^{TE} - V^{SO} - 9V^{TO}) \qquad 4$$

From the result of Table 1 of Fiase *et al.* (2006), and using equations (3) and (5), we derive the direct (V^D) and the exchange (V^{EX}) terms of our NOB effective interaction respectively to be:

$$V^D = -22272.37\frac{e^{-5r}}{5r} + 10208.88\frac{e^{-3r}}{3r} - 1896.94\frac{e^{-2r}}{2r} \quad 6$$

and

$$V^{EX} = 24124.32\frac{e^{-5r}}{5r} - 6372.5937\frac{e^{-3r}}{3r} + 178.66\frac{e^{-2r}}{2r} - 7.8474\frac{e^{-0.7072r}}{0.7072r} \quad 7$$

The Density Dependent factor of the New One Boson Nucleon-Nucleon Interaction

As is well known in nuclear matter calculations, the direct and exchange terms alone cannot reproduce the binding energy and the saturation condition of nuclear matter except a density dependence factor is included. To this end, the direct and exchange parts of our interaction are multiplied by a density dependent factor $f(\rho)$ adopted from Basu (2004), so that our density dependent interaction becomes

$$V(r,\rho) = V^{D(EX)}(r)f(\rho) \quad 8$$

where

$$f(\rho) = C(1+\beta e^{-\alpha\rho}) \quad 9$$

is the density dependent factor suitable for our interaction, and the constants C, β and a are density dependent parameters to be determined.

With this form of density dependence, the binding energy of nuclear matter per nucleon can be calculated as in Rashdan and Abdel-Karim (2002), using equation (10).

$$\epsilon = \frac{3\hbar^2 k_F^2}{10\text{m}} + \frac{f(\rho)\rho}{2}\{J_V + \int [j_i(k_F r)]^2 V^{EX}(r)d^3r\} \quad 10$$

where

$$J_V = \int V^D(r)d^3r \quad 11$$

and

$j_i(x) = 3\frac{j_1(x)}{x}$, while $j_k(x)$ is the k^{th} order spherical Bessel function.

Calculation of Nuclear Incompressibility:

The incompressibility, K_0 of the spin and isospin symmetric cold infinite nuclear matter which is a measure of the curvature of equation of state (EOS) at the saturation density is defined as in (Basu, 2004):

$$K_0 = K_F^2\frac{\partial^2\epsilon}{\partial K_F^2} = 9\frac{\partial^2\epsilon}{\partial\rho^2}|_{\rho=\rho_0} \quad 12$$

where the Fermi momentum K_F for the spin and isopin symmetric infinite nuclear matter is given by

$$K_F^3 = 1.5\pi^2\rho \quad 13$$

where ρ is the nucleonic density while ρ_0 is the saturation density for spin and isospin symmetric cold infinite nuclear matter.

In calculating the incompressibility, we used the zero-range pseudo potential to represent the exchange term (Basu,2004). In this case, the binding energy per nucleon is expressed as;

$$\epsilon = \frac{3\hbar^2 K_F^2}{10m} + \frac{f(\rho)\rho J_V}{2} = \frac{3\hbar^2 K_F^2}{10m} + \rho J_V C\frac{(1-\beta\rho^{\frac{2}{3}})}{2}, \quad 14$$

$$V(r,\rho,\epsilon) = [V^D(r) + J_{00}(\epsilon)\rho(r)]f(\rho), \quad 15$$

where $J_{00}(\in)$ is the zero–range pseudo potential representing the single nuclear exchange term and is taken to be:

$$J_{00}(\epsilon) = -276(1-0.005\epsilon) \quad \text{MeVfm}^3. \quad 16$$

The density dependent factor $f(\rho)$ is taken here to be of the form;

$$f(\rho) = C(1-\beta\rho^{2/3}) \quad 17$$

In (14), m is the nucleonic mass which is equal to 931.4943 MeV/c^2 and J_V represents the volume integral of the direct term of the NOB interaction supplemental by the zero – range pseudo potential having the form (18).

$$J_V = 4\pi\int V^D(r)r^2dr + J_{00}(\epsilon) \quad 18$$

Equation (14) can be differentiated with respect to ρ to yield

$$\frac{\partial\epsilon}{\partial\rho} = \frac{\hbar^2 K_F^2}{5m\rho} + \frac{J_V C(1-\frac{5}{3}\beta\rho^{\frac{2}{3}})}{2} \quad 19$$

The equilibrium density of nuclear matter was determined from the saturation condition $\rho\,\frac{\partial\epsilon}{\partial\rho} = 0$. Equations (14) and (19) with the saturation condition were solved simultaneously for fixed values of the saturation energy per nucleon ϵ_o and the saturation density $\boldsymbol{\rho}_0$ of the standard nuclear matter, to obtain the values of density dependent parameters β and C given respectively as

$$\beta = \frac{[(1-p)\rho_0^{-\frac{2}{3}}]}{[3-\frac{5}{3}p]}, \quad 20$$

$$C = \frac{-[2\hbar^2 K_{F_0}^2]}{[5mJ_v\rho_0\left(1-\left(\frac{5}{3}\right)\beta\rho_0^{\frac{2}{3}}\right)]} \quad 21$$

where

$$P = \frac{10m\epsilon_0}{\hbar^2 K_{F_0}^2} \quad 22$$

and

$$K_F = (1.5\pi^2\rho_0)^{1/3} \qquad 23$$

It should be noted that the density dependent parameter β obtained by this method depends only on the saturation energy per nucleon ϵ_o, saturation density ρ_0 but not on the parameters of the NOB interaction while the other density dependent parameter C depends on the parameter of the NOB interaction also through the volume integral J_V The energy per nucleon can thus be rewritten as:

$$\epsilon = \frac{3\hbar^2 k_F^2}{10m} - \left(\frac{\rho}{\rho_0}\right)\frac{[\hbar^2 k_{F0}^2(1-\beta\rho^{2/3})]}{\left[5m(1-\left(\frac{5}{3}\right)\beta\rho_0^{2/3})\right]} \qquad 24$$

The incompressibility, K_0 defined by equation (12) can then be evaluated using equations (20), (21), (22) and (23) to give

$$K_0 = -\left(\frac{3\hbar^2 k_{F0}^2}{5m}\right) - 5J_V C\beta\rho_0^{5/3} \qquad 25$$

Numerical Results of Pressure, Energy Density and Velocity of Sound

The pressure p and energy density ε of nuclear matter are calculated from (Chowdhury and Basu, 2006);

$$P = \rho^2\frac{\partial\epsilon}{\partial\rho} = \frac{\rho\hbar^2 k_F^2}{5m} + \rho^2 J_V C\frac{[1-\left(\frac{5}{3}\right)\beta\rho^{2/3}]}{2} \qquad 26$$

and

$$\varepsilon = \rho(\epsilon + mc^2) = \rho[\left(\frac{3\hbar^2 k_F^2}{10m}\right) + \rho J_V C\frac{(1-\beta\rho^{2/3})}{2} + mc^2] \qquad 27$$

respectively.

The velocity V_s of sound in standard nuclear matter is also expressed as

$$\frac{V_s}{c} = \sqrt{\frac{\partial P}{\partial\varepsilon}} = \sqrt{\left[2\rho\frac{\partial\epsilon}{\partial\rho} - \frac{\hbar^2 k_F^2}{15m} - \frac{5}{9}J_V C\beta\rho^{5/3}\right] / \left[\epsilon + mc^2 + \rho\frac{\partial\epsilon}{\partial\rho}\right]} \qquad 28$$

Results and Discussion

Fig.1 gives the graph of the calculated values of the binding energy per nucleon for infinite nuclear matter plotted with the requirement that the nuclear binding energy be reproduced at the correct nuclear matter density. From the graph, it can be seen that, the binding energy per nucleon of -16 MeV was reproduced at the nuclear density approximately $0.17 fm^{-3}$.

The theoretical estimate of the compression modulus (K_0) of infinite symmetric nuclear matter (SNM) obtained from the present approach using our NOB interaction ranges from 303 to 309 MeV. The present result of K_0 is found to be in perfect agreement with the work of Basu (2004), in which the value of K_0 ranges from 304 to 310MeV. The theoretical estimate of K_0 by infinite nuclear matter model (INM)by Satpathy *et al* (1988), claims a well-defined and stable value of $K_0 = 288 \pm 20$ MeV and the present estimate is in good agreement with the value of K_0 obtained by INM which rules out any values lower than 200 MeV. The present estimate for the incompressibility K_0 of the infinite SNM is also in good agreement with the experimental value of K= 300 ± 25 MeV obtained from the giant monopole resonance (GMR) (Shama, 1988), and the experimental determination of K_0 based on the production of hard photons in heavy ion collision which led to experimental estimate of $K_0 = 290 \pm 50$ MeV (Schutz 1996). The theoretical description of nuclear matter based on mean – field calculation using DDM3Y effective interaction however yields a value of about 270 MeV for nuclear incompressibility (Basu, 2014). Also, the equation of state based on both Paris and Reid CDM3Y interaction provide saturation incompressibility of symmetric nuclear matter in the range of 220 – 270 MeV (Seif and Basu, 2014).

In Table 2, the theoretical estimates of pressure p and velocity of sound V_s of standard nuclear matter using our NOB NN interaction have been presented as functions of nucleonic density ρ and energy density ε using the usual value of 0.005/MeV for the parameter *a* of energy dependence of the zero- range pseudo-Potential. As for any other non- relativistic EOS, present equation of state also surfers from super – luminosity at very high densities. According to the present calculations, the velocity of sound becomes imaginary for $\rho \leq 0.1 fm^{-3}$ and exceeds the velocity of sound at $\rho \geq 5.3\rho_0$. This is in excellent agreement with the EOS obtained using v_{14}+TNI by Friedman and Pandharipande (1981), which also resulted in sound velocity becoming imaginary at the same nuclear density and super-luminous at about the same nuclear density.

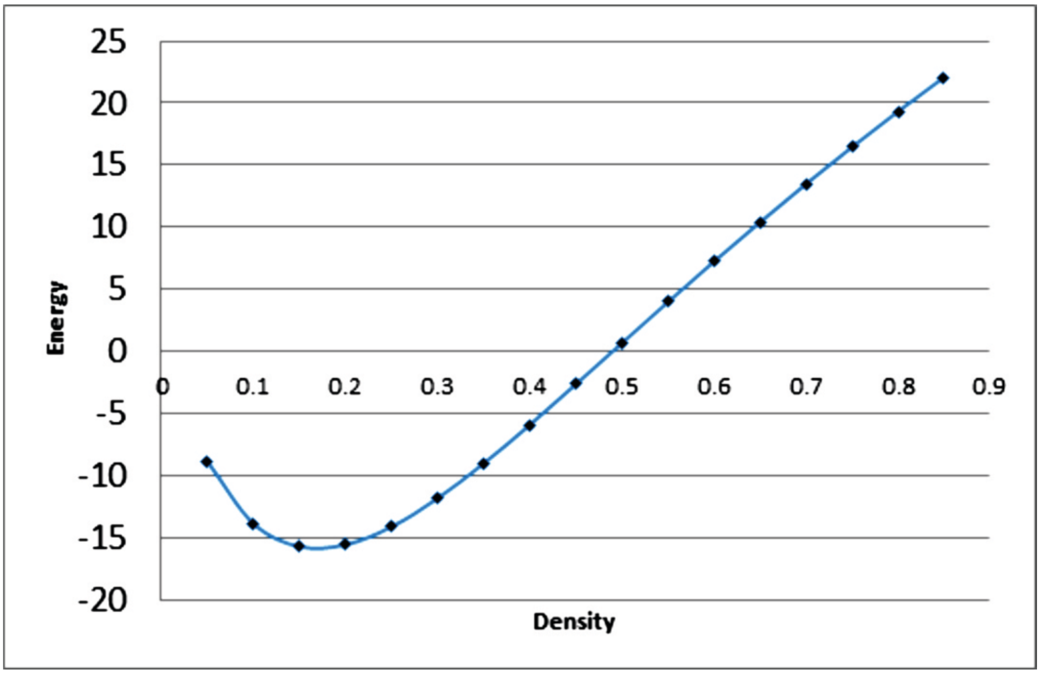

Fig. 1, the graph of the calculated values of the binding energy per nucleon for infinite nuclear matter against densities.

Table1: The result of the present calculation of k_0 compared with those of Basu (2004).

$\rho_0(fm^{-3}$	$\beta(fm^2)$	**C (Basu, 2004)**	**C present**	**K_0(MeV) (Basu, 2004)**	**K_0(MeV) Present**
0.170	1.551	1.98	1.14	309.6	308.6
0.165	1.586	2.02	1.17	308.2	307.7
0.160	1.624	2.07	1.20	306.9	306.8
0.155	1.664	2.11	1.22	305.5	304.8
0.150	1.705	2.16	1.25	304.0	303.5

TABLE 2: Summary of results; calculated energy per nucleon ϵ pressure P, energy density ε and velocity of sound V_s of nuclear matter as functions of nucleon density ρ, using saturation energy per nucleon equal to – 15.260 MeV and saturation density $\rho_0 = 0.1533$ fm^{-3} for standard nuclear matter.

ρ [fm^{-3}]	ρ/ρ_0 [Mev]	ϵ [Mevfm^{-3}]	P [Mevfm^{-3}]	ε	V_s in units of c
0.01	0.065	-0.750	-0.0115	9.322	Imaginary
0.10	0.652	-13.232	-0.519	92.036	Imaginary
0.20	1.305	-13.760	2.638	184.022	0.289
0.30	1.957	-1.131	15.550	279.783	0.471
0.40	2.609	23.403	43.3000	382.666	0.623
0.50	3.262	58.933	90.530	495.698	0.746
0.60	3.914	104.771	161.500	621.684	0.846
0.70	4.566	160.372	260.100	763.258	0.9269
0.80	5.219	225.290	390.100	922.921	0.991
0.90	5.871	299.151	555.100	1103.061	1.043
1.00	6.523	381.633	758.600	1305.974	1.085

Conclusion.

A density – dependent new one boson interaction has been derived from the variational technique. One of the stringent tests an effective interaction must pass among other tests is to reproduce the binding energy of nuclear matter of -16 MeV at the nuclear matter saturation density of 0.17 fm^3. This is achieved as our NOB –

interaction reproduces the binding energy of approximately -16 MeV at the correct nuclear matter saturation density. A study of incompressibility of symmetric cold infinite nuclear matter using the direct term of our NOB interaction and the BD – density dependent factor with the zero – range psedopotential representing the single nuclear exchange term was also carry out. The nuclear matter density was varied from 0.15 to 0.17 fm^{-3}. The values for nuclear incompressibility obtained at different nuclear matter densities agree reasonably with those obtained by Basu (2004) using the M3Y – Reid interaction.

The values of pressure, energy density and velocity of standard nuclear matter obtained in our present calculations also surfers from super – luminosity as with any other non – relativistic equation of state. Our results shows that, the velocity of sound becomes imaginary for $\rho \leq 0.1 fm^{-1}$ and exceeds the velocity of sound at $\rho \geq 5.3 \rho_0$, which is in excellent agreement with (Chawdhury, *et al.,* 2006).

A very important milestone of the present work is that, a new interaction for nuclear matter study based on LOCV technique has been developed. This interaction have proven to be effective haven satisfy the basic test for an effective interaction. The fact that this interaction is new, we suggest that it should be applied for more nuclear matter calculations and optical model analysis to further investigate its performance.

References

Basu, D. N. (2004).Nuclear incompressibility using the density – dependent M3Y effective interaction. *Journal of Physics G: Nuclear and particle physics,*30: 7 –11.

Basu, D. N. (2014). Effective interaction from nuclear reaction to neutron stars. *PRAMANA Journal of Physics*, 82: 809 - 822.

Bertsch, G., J, Borysowicz., H, Mcmanus and W. G. Love. (1977). Interactions inelastic scattering derived from realistic potentials. *Nuclear Physics A*, 284: 399 – 419.

Chaudhuri, A.K. (1986). A density dependent interaction in the folded alpha – nucleus potential. *Nuclear physics A*, 449: 243-250.

Chowdhury, P. R. and D, N. Basu. (2006). Nuclear matter and re – evaluated coefficients of liquid drop model. *Acta Physica Polonnica B*, 37: 1833 – 1845.

Dortmans, P.J. and K, Amos. (1994). Density – dependent effective interactions. *Physical Review C,* 49:1309 – 1314.

Fiase, J. O., K, R. S. Devan and A, Hosaka. (2002). Mass dependent of M3Y – type interaction and effects of tensor correlations. Physical Review C, 66: 014004

Fiase, J. O., F, Gbaorun andL, K. Sharma. (2006). New – One – Boson Exchange Potential Functions. *Journal of Nigerian Association of Mathematical Physics,* 10: 5 – 10.

Fiase, J. O. and F, Gbaorun. (2011). Binding energy and compression modulus of infinite nuclear matter derived from variational calculation. *Journal of Nigerian Association of Mathematical Physics*, 19: 615 – 618.

Fredmam, B. and V, R. pandharipande. (1981). Hot and cold nuclear and neutron matter. *Nuclear Physics A*, 361:502 – 520

Hosaka, A., K, R. S. Kuboand H, Toki. (1985). G – Matrix effective interaction with Paris potential. *Nuclear Physics A*, 444: 57-65.

Misicu, S. (2007). Nuclear matter equation of state for the practitioner. Romanian Reports in Physics, 59(1): 1137-1140.

Rashdan, M. and T, A. Abdel-Karim. (2002). Test of D3Y effective interaction for calculating ion –ion optical potential. Physca Scripta A, 65: 133 -138

Satpathy, L. U., V,S. Maheswair and R, C. Nayak. (1988). Masses of Atomic nuclei in the infinite nuclear matter model. Physics Report, 319: 89- 95

Schutz, Y. (1996). The role of incompressibility in the production of hard photons in heavy – ion collisions. *Nuclear Physics A*, 599: 97-105.

Seif, W. M. and D, N. Basu. (2014). Higher – order symmetry energy of nuclear matter and the inner edge of neutron star crusts. Physical Review C, 89: 028801 –

Shama, M. M., W, T. A. Borghols., S. Brandenbury, S, Crona., A, V.Woude and M, N. Harakeh. (1988). Giant monopole resonance in Sn and Sm nuclei and the compressibility of nuclear matter. Physical Review C, 38: 2562 – 2566.